SENIOR TWO WORK.

INEQUALITIESAND REGIONS

EQUATIONS AND INEQUALITIES

Supposing we have a class with 70 students, if n stands for the number of students in such class then n=70.
The two side are equal and therefore n=70 is an equation.

But if the class before this one has fewer students then we say that n is less than 70.
i.e. $n<70$.

If the one after this class has more students then we say that n is greater than 70.
i.e. $n>70$

∴ $n>70$ and $n>70$ are inequalities or inequations.

We use inequalities to compare two or more quantities.
A weighing scale can be used to compare the weight of items.

Inequality statements like; greater than, less than, greater than or equal to, less than or equal to, may be substituted by inequality symbols.

Inequality symbols

$>$ stands for more than (greater than)

$<$ stands for less than

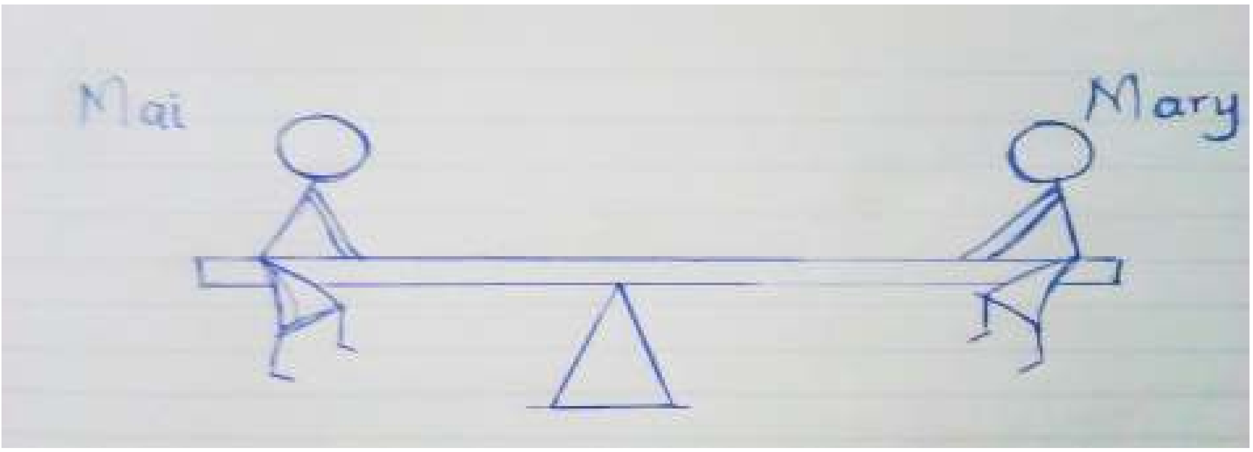
$\geq$ stands for equal to or greater than.

$\leq$Stands for equal to or less than.

Example:1

In each of the following diagrams, compare the two quantities and use an appropriate inequality sentence or symbol to represent them.

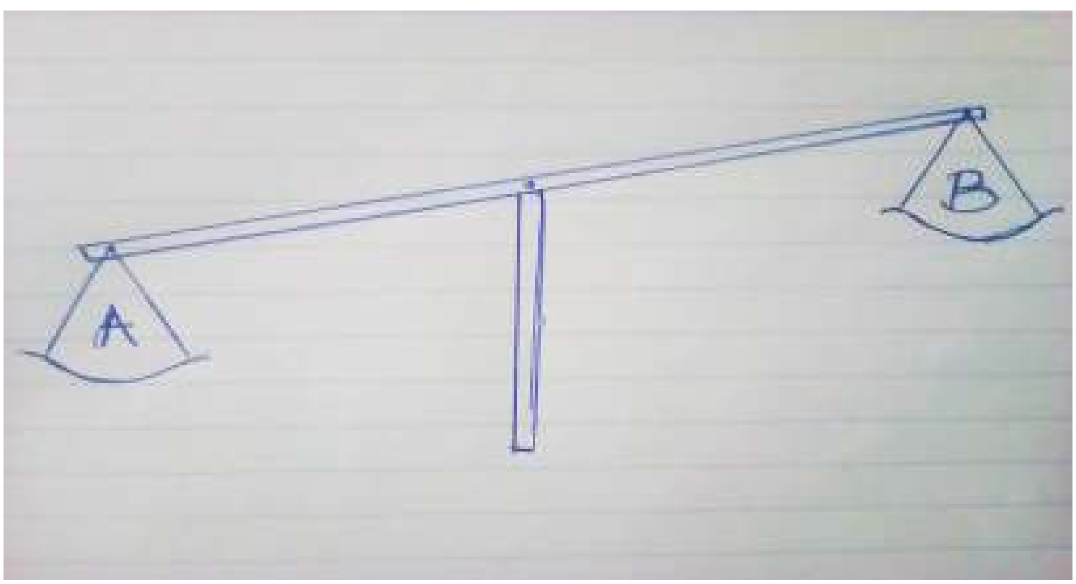
(a)



Mai weighs as much as Mary.

Mai’s weight is equal to Mary’s weight.

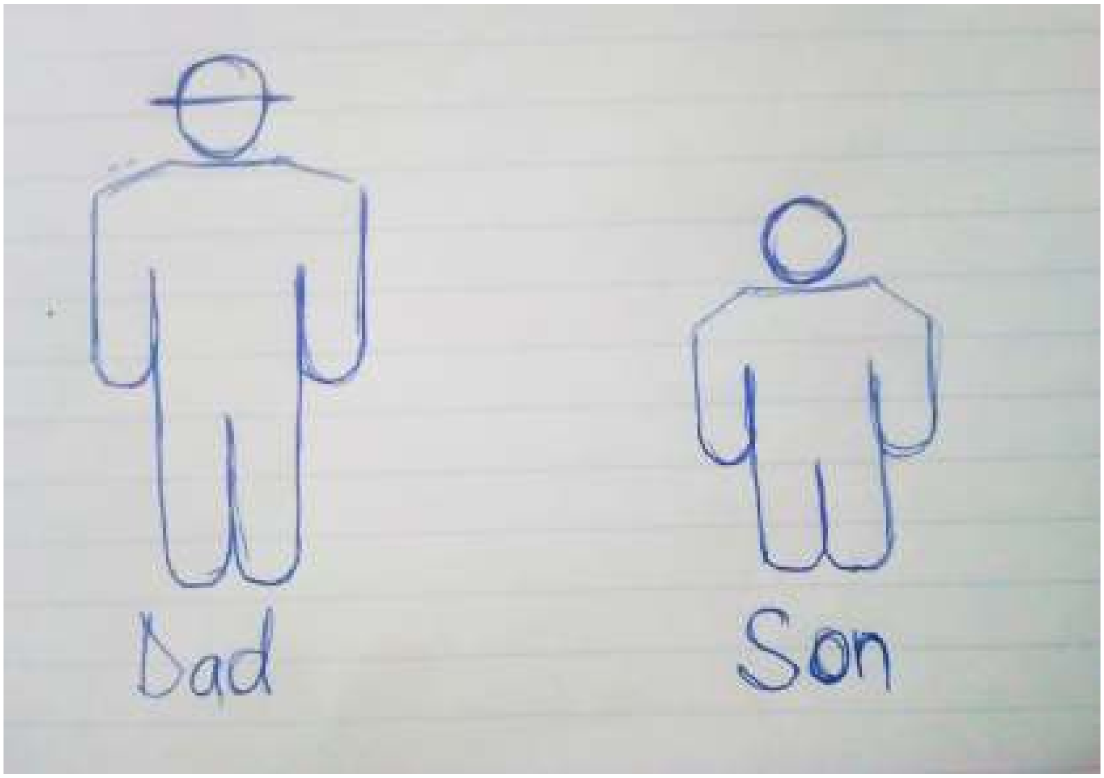
(b)



A is heavier than B

B is lighter than A

(c)



Dad is taller than son

Son is shorter than Dad.

<u>Example:2</u>

Write the following using inequality sign.

(a) $t$is less than 4.   ↔ (t < 4)

(b) x is greater or equal to 7.↔ ($x \leq 7$)

(c) $x$is more than 17.↔($x > 17$)

(d) $x$is greater than 4 and less or equal to 7. ↔ ($4 < x \leq 7$)

(e) y is less than 4 and equal to or greater than -5.

↔($-5 < y \leq 4$)

Sets of numbers

We can write the set of natural numbers x such that x is less than or equal to 7. This is denoted by$\{x: x\epsilon N, x \leq 7\}$

Therefore $\{x: x \in N, x \leq 7\} = \{1,2,3,4,5,6,7\}$

The set of natural numbers y such that y is greater than 4 is donated by $\{y: y \in N, y > 4\}$

The natural numbers that are greater than 4 are

$\{5,6,7,8,9,10,11,12,13,14,15 \cdots\}$

The list is infinity.

Example 3:

List members of the following sets of natural numbers.

(a) $\{x: x \in N, x < 5\}$

(b) $\{x: x \in N, 5 < x < 10\}$

(c) $\{x: x \in N, 4 \leq x < 8\}$

(d) $\{e: e\ is\ even\ and\ 3 < e \leq 12\}$

(e) $\{y: y\ is\ odd, 5 \leq y \leq 11\}$

(f) $\{n: n\ is\ a\ factor\ of\ 30\}$

Solutions:

(a) The natural numbers less than 5 are 1,2,3,4

$$\therefore \{x: x \in N, x < 5\} = \{1,2,3,4\}$$

(b) The natural numbers between 5 and 10 are 6,7,8 and 9.

$$\therefore \{x: x \in N, 5 < x < 10\} = \{6,7,8,9\}.$$

(c) $\{x: x \in N, 4 \leq x < 8\} = \{4,5,6,7\}$.

(d) The even numbers between 3 up to 12 are 4,6,8,10 and 12
$\therefore$ The list $\{4,6,8,10,12\}$.

(e) The odd numbers from 5 up to 11 are 5,7,9 and 11.
The list is $\{5,7,9,11\}$.

(f) Factors of 30 are; 1 and 30, 2 and 15, 3 and 10, 5 and 6.
The list is $\{1,2,3,5,6,10,15,30\}$.

<u>Exercise:</u>

1) Write the following using inequality signs.

(a) $x$is less than 6.

(b) y is greater than or equal to -3.

(c) $p$is less than 4.

(d) t is greater than 4 and less or equal to 7.

(e) n is greater than or equal to -3 and less than or equal to 10.

2) Write the following inequalities in words.

(a) $p < 6$
(b) $-3 \leq d \leq 21$
(c) $3 < y \leq 6$
(d) $e > 5$
(e) $-2 \leq r < 5$
(f)$7 < y$

3) List the members of the following sets.

(a) $\{x: x \in N, x < 8\}$
(b) $\{x: x \in N, 8 \leq x < 12\}$

(c) $\{y : y \text{ is a multiple of } 4\}$

(d) $\{p : p \text{ is prime}, p < 25\}$

PRESENTING INEQUALITIES ON A NUMBER LINE.

We can represent inequalities on a number line bearing in mind that all numbers to the left of a number are less than that number. Similarly, all the numbers to the right of a number are larger than that number .

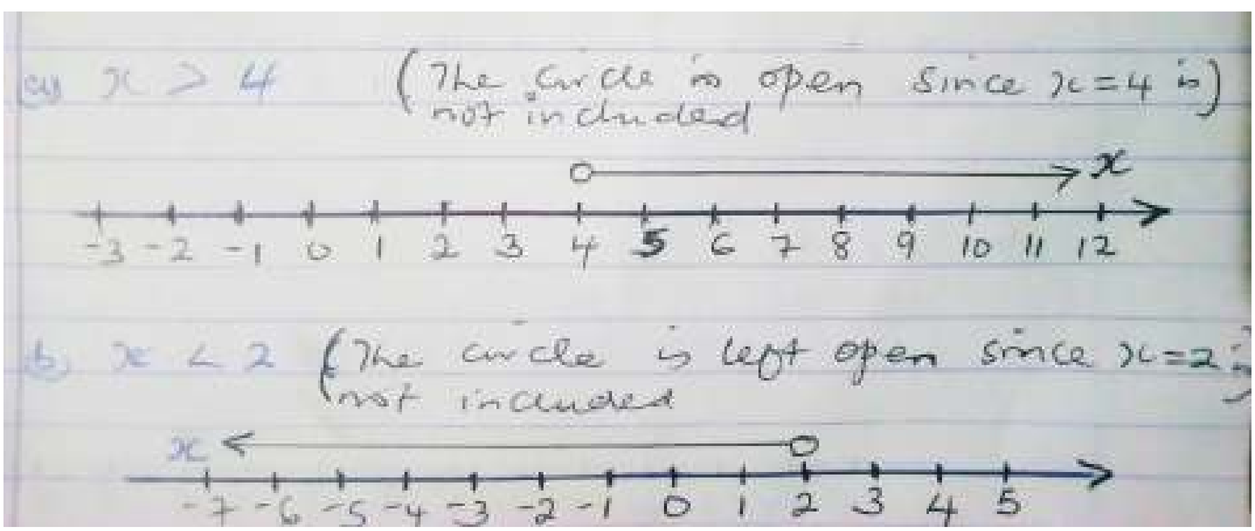
We draw a small circle,o, at the starting point.

The circle ,o is left open if the starting point does not satisfy the inequality (i.e. for < or >).

The circle,o, is filled-in if the starting point satisfies the inequality (i.e. for ≤or ≥). .

Example:

Show each of the following inequalities on a number line.



c) $x \leq 5$ ($the\ circle\ is\ filled - in\ since\ x = is\ included$)

Note: Each of the inequalities in (a), (b) and (c) has one boundary (end).

The inequalities below have both ends.

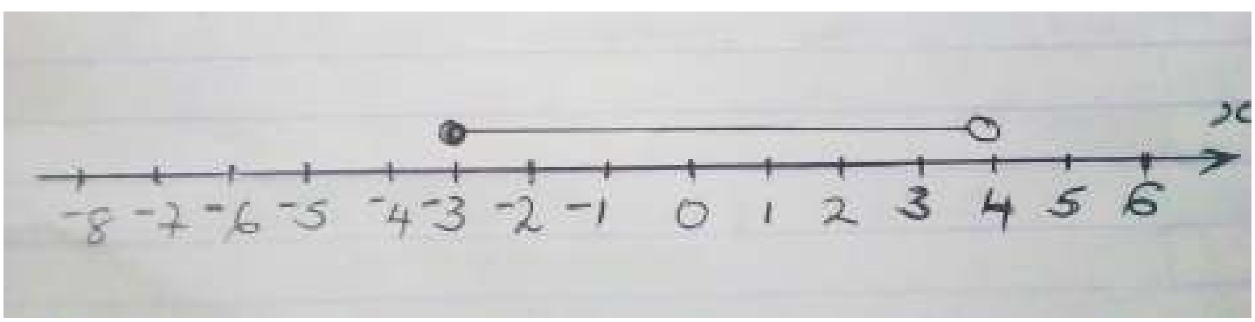
Show the on a number line

(a) $-3 \leq x < 4$

(b) $-2 < y \leq 5$

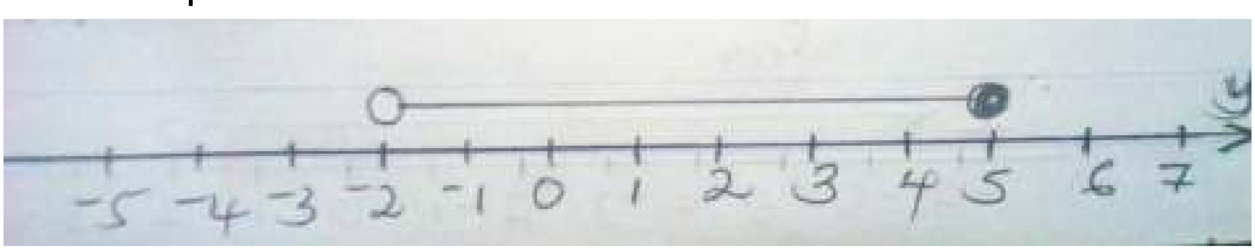
a) $-3 \leq x < 4$

We use a filled-in circle over -3 and an open circle over 4.



b) $-2 < y \leq 5$

We use open circle over -2 and a filled-in circle over 5.



Solving inequalities is similar to solving equations

Compare solving an equation and an inequality.

1) i) $3x = 9$and ii) $3x < 9$

$$\frac{3x}{3} = \frac{9}{3}\frac{3x}{3} < 9$$

$$x = 3x < 3$$

2) i) $3x + 2 = -4$ and ii) $3x + 2 > -4$

$$3x + 2 - 2 = -4 - 23x + 2 > -4 - 2$$

$$\frac{3x}{3} = \frac{-6}{3}\frac{3x}{3} > \frac{-6}{3}$$

$$x = -2x > -2$$

Now solve:

3) $5x + 2 \leq 12$

$$5x + 2 - 2 \leq 12 - 2$$

$$x \leq 2$$

4) $3x - 5 > -2$

$$3x - 5 + 5 > -2 + 5$$

$$\frac{3x}{3} > \frac{3}{3}$$

$$x > 1$$

All the steps taken when solving an inequality are the same as those taken when solving an equation except if you have to divide or multiply the inequality by a negative number.

In this case the order changes ie$\geq$ changes to$\leq$ and $\leq$changes to$\geq$

5) Solve:

$$3 - 2x \geq 9$$
$$3 - 3 - 2x \geq 9 - 3$$
$$\frac{-2x}{-2} \geq \frac{6}{-2}$$
$$x \leq -3$$

6) Solve:

$$9 - x \leq 3 + x$$
$$9 - x - x \leq 3 + x - x$$
$$9 - 2x \leq 3$$
$$9 - 9 - 2x \leq 3 - 9$$
$$\frac{-2x}{-2} \leq \frac{-6}{-2}$$
$$x \geq 3$$

7) Solve:

$$7 < 3x + 1 < 19$$
$$7 - 1 < 3x + 1 - 1 < 19 - 1$$
$$\frac{6}{3} < \frac{3x}{3} < \frac{18}{3}$$
$$2 < x < 6$$

Note: After solving the inequalities, illustration on the number line is done as before.

<u>Exercise:</u>

1.Solve the following inequalities and illustrate the solution on a number line.

a) $2x \leq 4$.

b) $-8 < 4x < 12$.

c) $3y + 10 > 25$.

d)$3x - 4 < 5$.

e) $3x + 2 \leq -4$

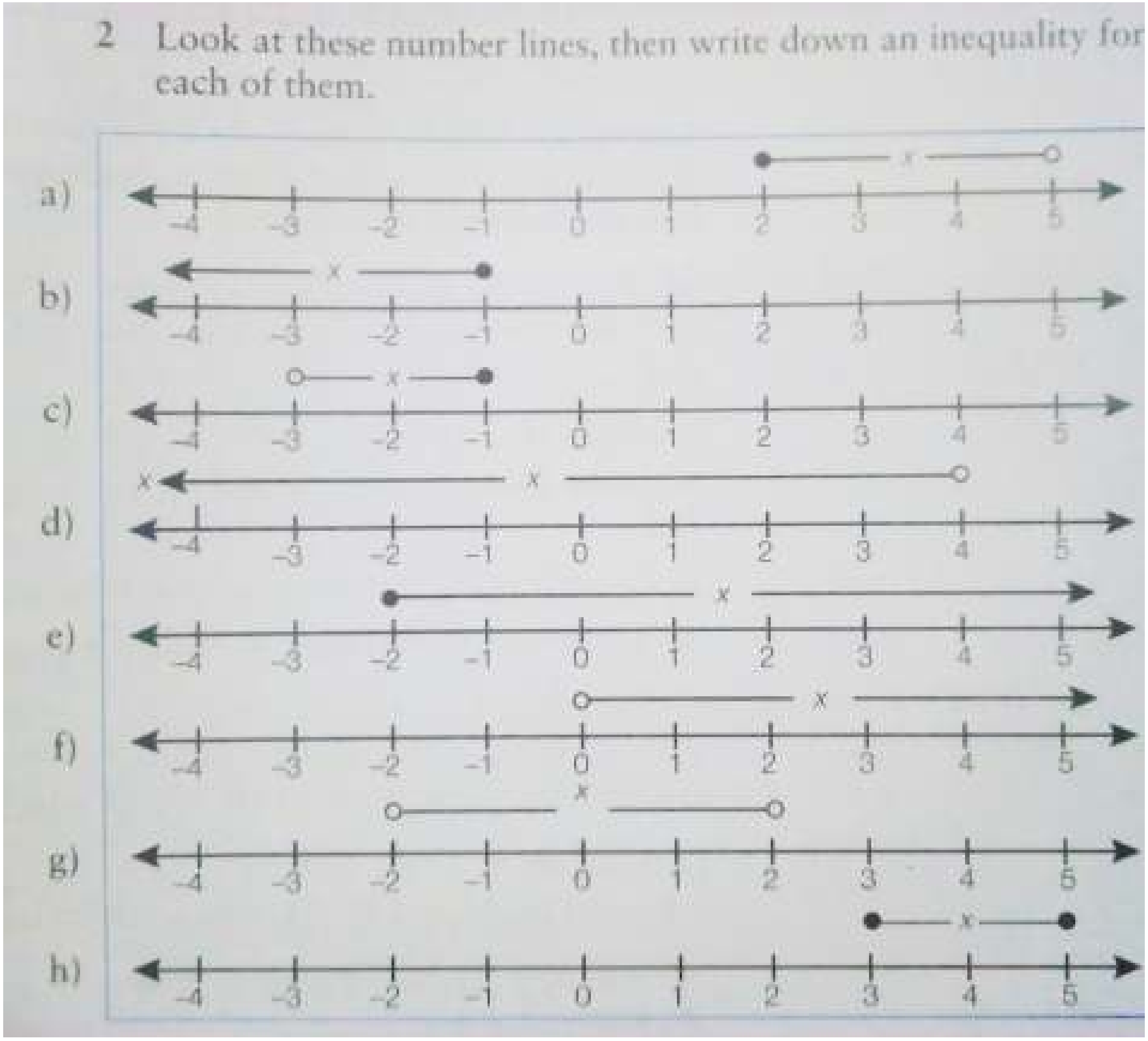
f) $2(x - 1) > x - 1$.

g) $5x + 3 < -11 - 2x$.

h) $7 < 3x + 1 < 19$.

i) $5 - 2x > x - 4$.

j) $x + 5 < 3x - 7$


2 Look at these number lines, then write down an inequality for each of them.

Showing inequalities on a gragh: Regions

We can show an inequality on an $x - y$ plane (Cartesian plane)

We look at the inequality as if it is an equation and plot the line corresponding to it. This is called the boundary line.

The boundary line divides the $x - y$ plane into two regions; the wanted region, and the unwanted region.

If the inequality sign is < or >, we draw a dotted line or a broken line.

If the inequality sign is $\leq$ or$\geq$, we draw a continuous or solid line.

Example:

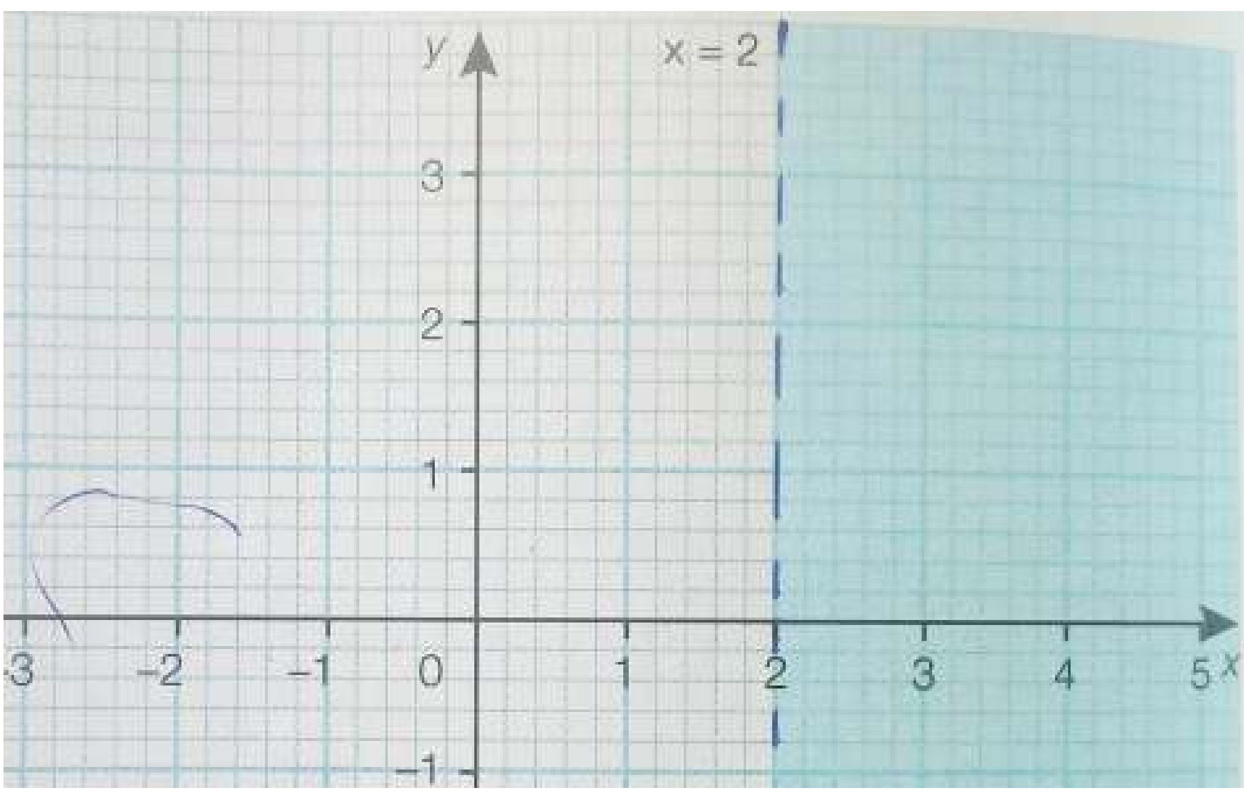
Show the region $x < 2$ on a graph by shading out the unwanted region.

Solution:

We draw the line $x = 2$ with a dotted line indicating that the points on the line do not satisfy the inequality and are not included. Since values of x must be less than 2.

The line $x = 2$ is the line parallel to the y-axis that goes through $x = 2$.

We shade the region where the x values are greater than 2



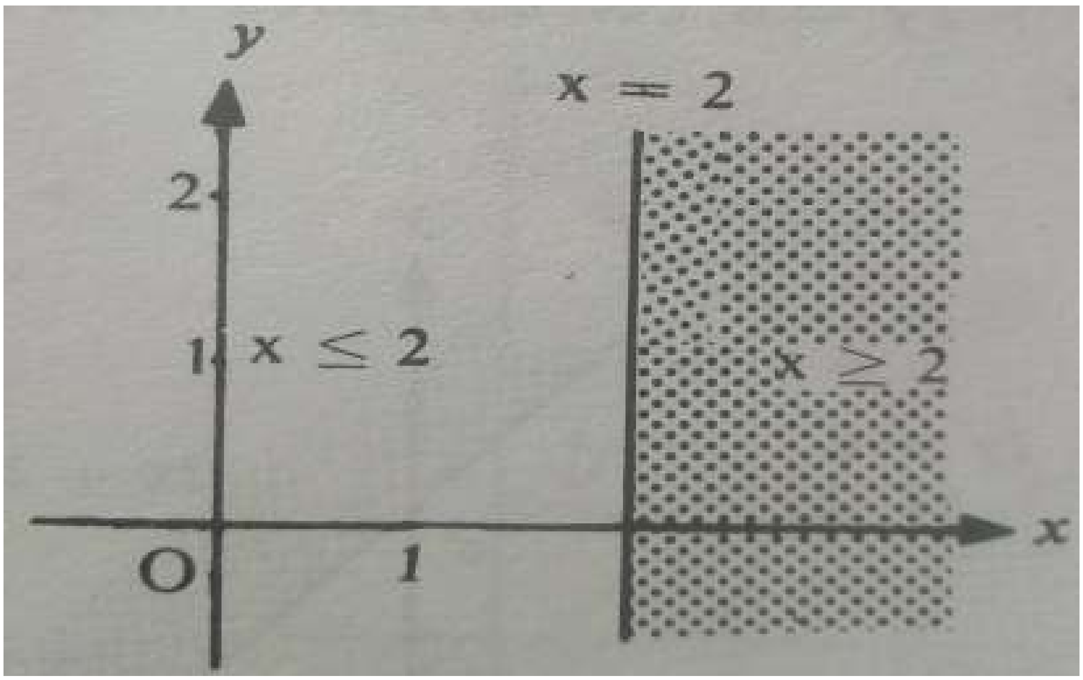
Example:

Show the region $x \leq 2$ on the graph.

Solution

Inequalities of the type $\leq$ and $\geq$ include the boundary line.

For the case of $x \leq 2$ the boundary line $x = 2$ is a continuous line (solid line).



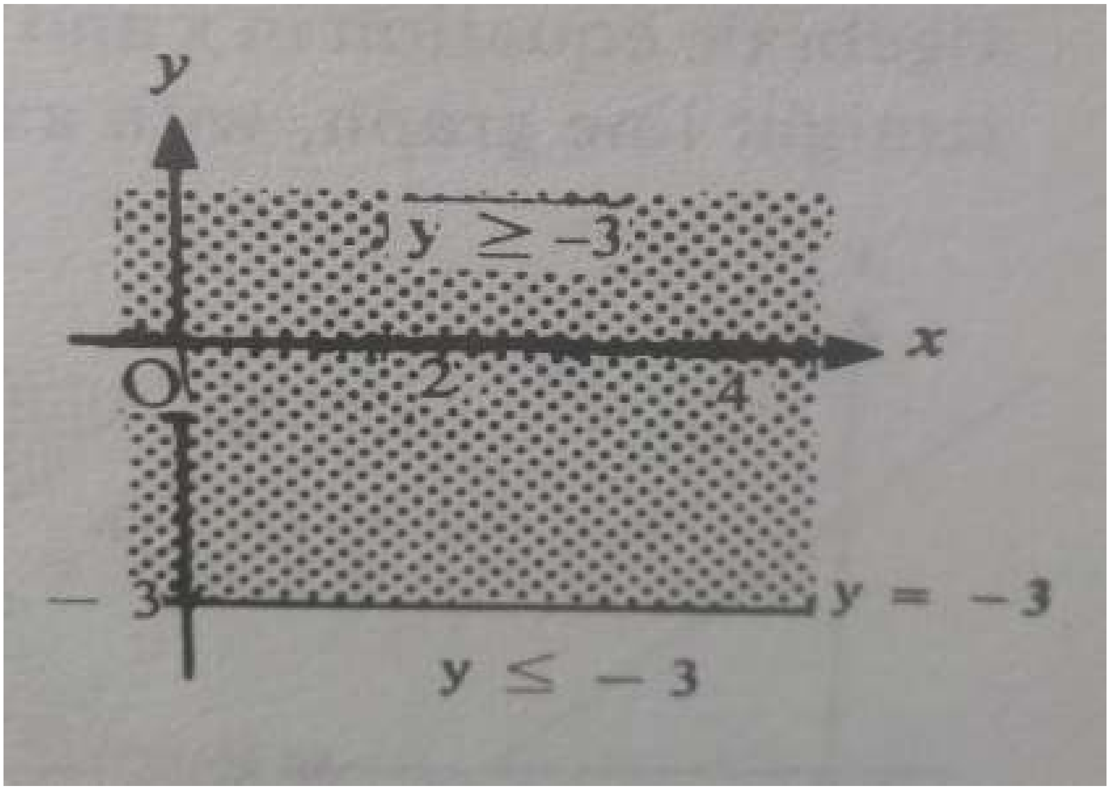
Example:

Show the region $y \leq -3$

Solution

The boundary line $y = -3$ is drawn solid.

The unwanted region is the region where the y values are greater than -3.



<u>Example</u>

Show the region for which $y \geq x + 3$

<u>Solution</u>

This inequality shows the relationship between two variables x and y. We need to draw the boundary line $y = x + 3$ and it is a solid line.

In order to draw this line, we need the points that are on it.

When $x = 0$, $y = 3$, hence the point (0,3) lies on the line.
When $y = 0, x = -3$, hence point (-3,1) lies on the line.
When $x = 1, y = 4$, hence the point (1,4) lies on the line.

x	**0**	**3**	**1**
y	3	0	4

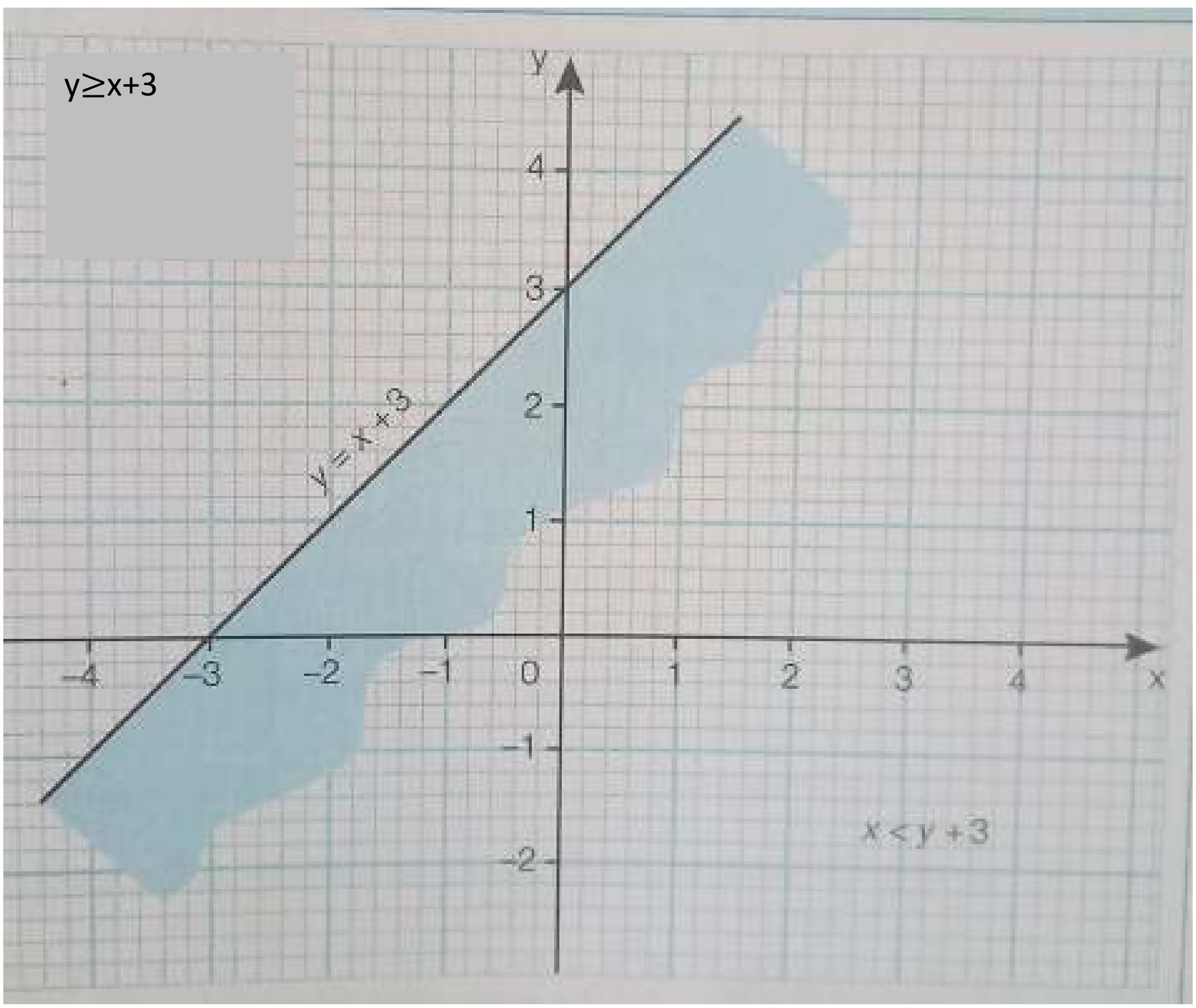
After drawing the boundary line, we select a point that is not on the line and use it to investigate the regions

$y > x + 3$

At (0,0), 0 is not greater than 0+3

0 is not greater than 3

∴ (0,0) is in the unwanted region and we shade the region which contains point (0,0).



Example:

Show the region for which $3x + 2y < 6$, by shading out the unwanted region.

Solution

We first need to draw boundary line $3x + 2y = 6$.

This line will be dotted.

Points on the line are obtained.

When $x = 0, y = 3 \quad (0,3)$

When $y = 0, x = 2 \quad (2,0)$

When $x = 1, y = 1.5\ (1,1.5)$

$\boldsymbol{x}$	0	2	1
$\boldsymbol{y}$	3	0	1.5

When the line is drawn, we investigate the wanted region.

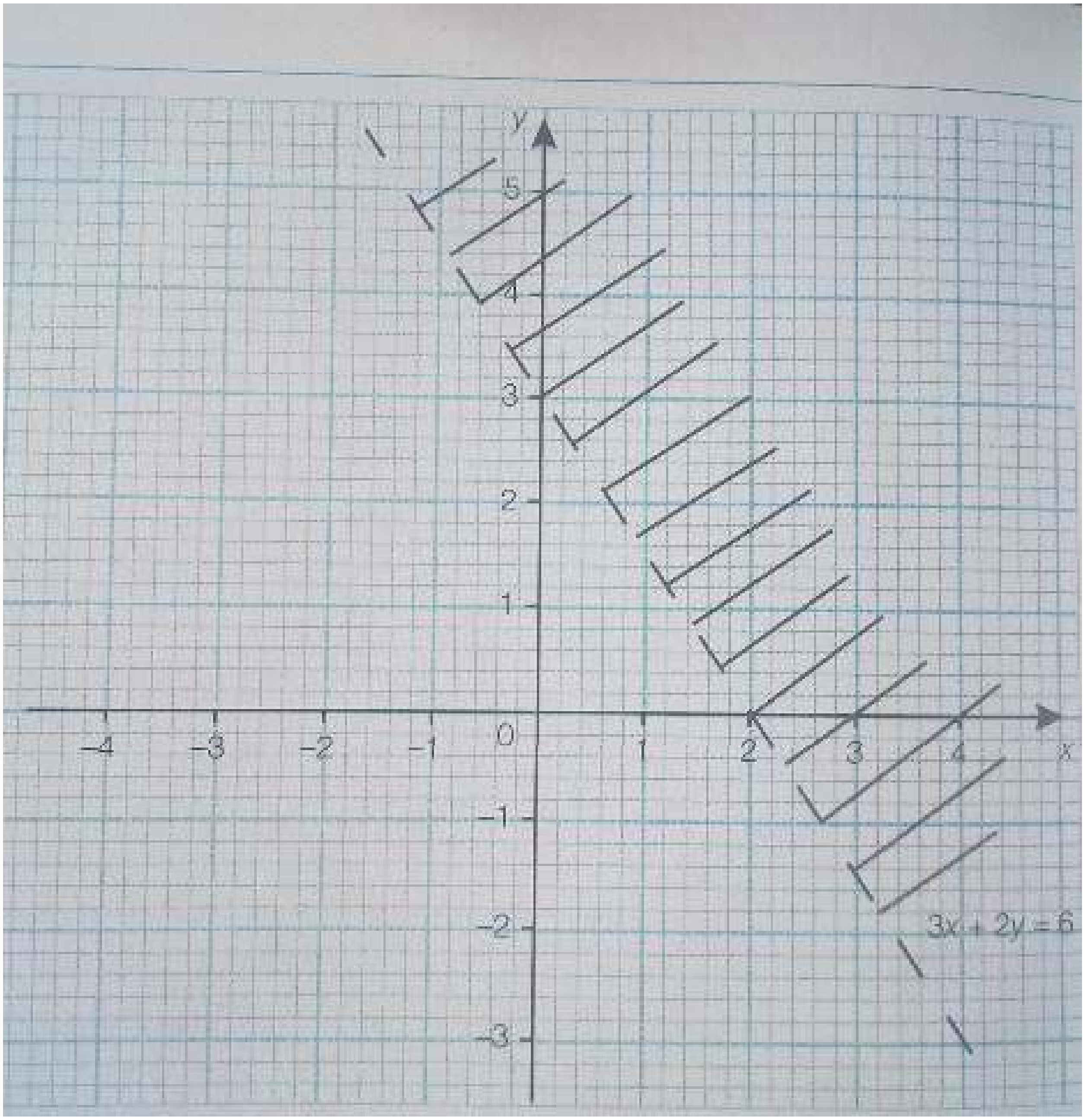
The inequality is $3x + 2y < 6$.

At (1,1), 3(1)+2(1)<6

3+2<6

5<6

Since (1,1) satisfies the given inequality, then, point (1,1) lies in the wanted region.



<u>Example:</u>

Show the region in which$3x + 4y < 12$ by shading the unwanted region.

<u>Solution:</u>

Boundary line is $3x + 4y = 12$ and is to be dotted.

Points on the line;

x	0	4	2
y	3	0	1.5

Draw the line and chose a point to investigate the wanted region

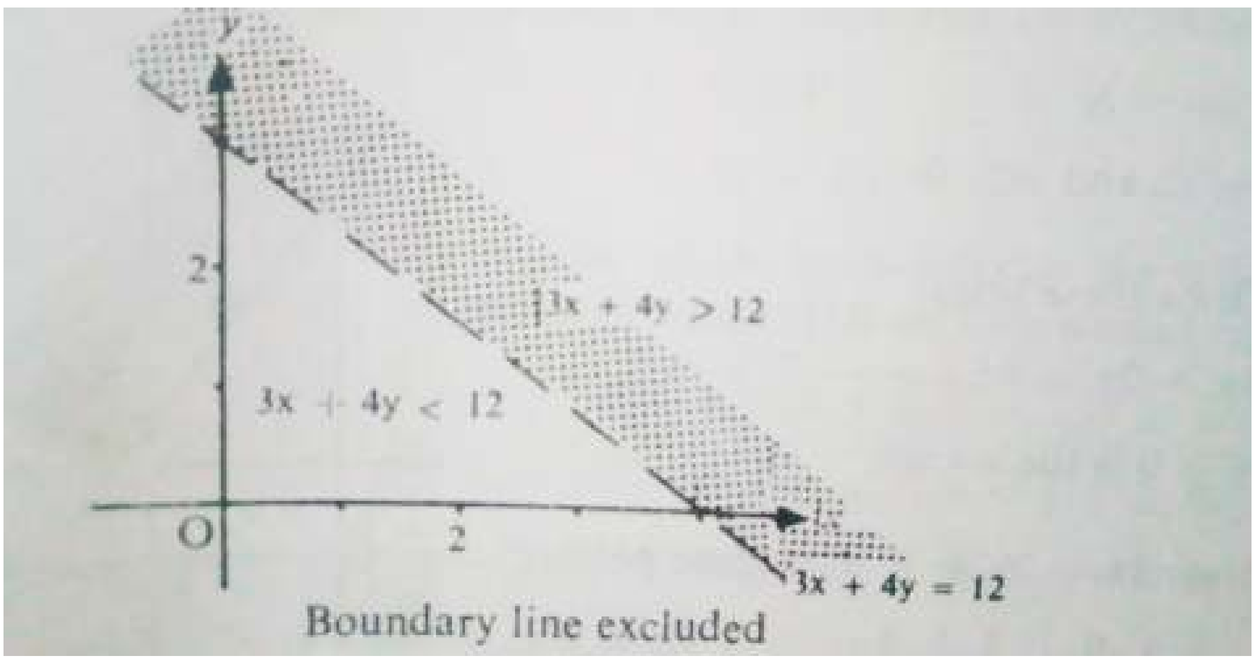
$$3x + 4y < 12$$

At (0,0)

0+0<12

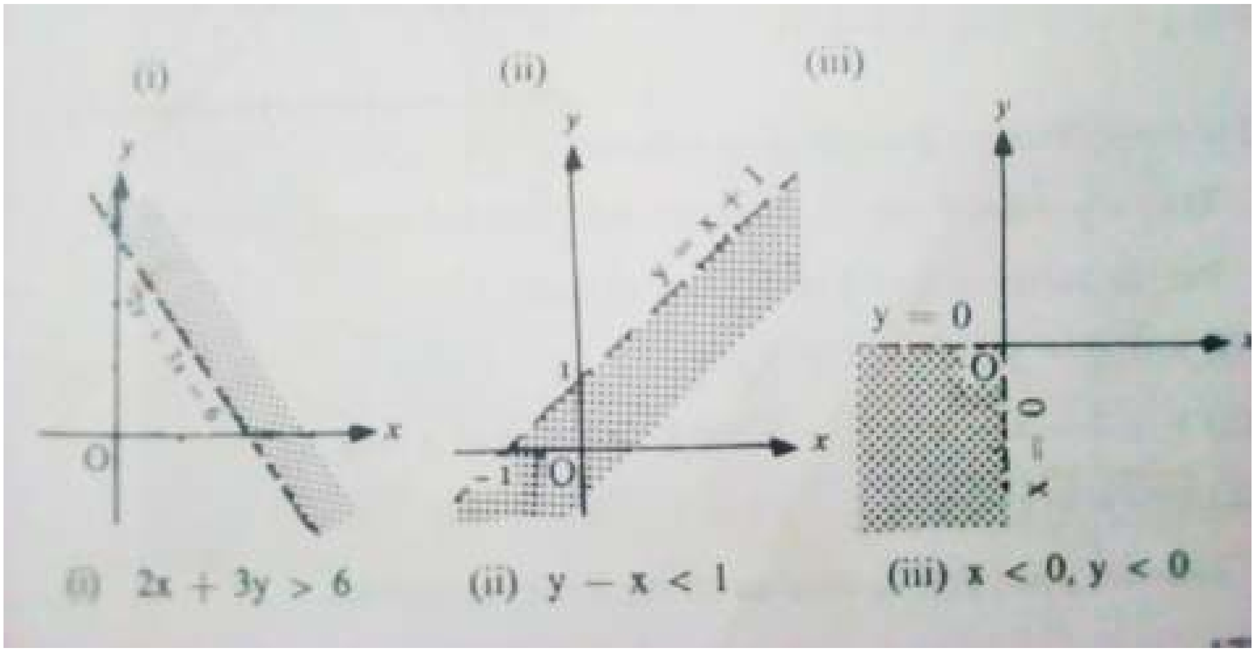
0<12

Since point (0,0) satisfies the given inequality then it is in the wanted region.



Example:

State the inequality satisfied by the shaded region.



Solution to a set of simultaneous inequalities.

To be continued…

STAY HOME

STAY SAFE