INEQUALITIES AND REGIONS

Summary:

1. *The following symbols are used when dealing with inequalities* **<, ≤, >** *and ≥*

2. *The inequality symbol reverses when you multiply or divide an inequality by a negative number*

3. *To represent an inequality on a number line, use an open circle for* **< or >** *symbol and in case of ≤ or ≥, use a closed circle.*

4. *Integers in the range of a given inequality are called integral values*

5. *The inequality* $2 > x$ *is the same as* $x < 2$ *and* $4 < x$ *is the same as* $x > 4$.

EXAMPLES:

1. *Represent each of the following inequalities on a number line:*

(i) $x \geq -2$ (ii) $x > 3$ (iii) $x \leq 2$ (iv) $x < 4$ (v) $-1 \leq x \leq 5$

(vi) $-2 < x < 3$ (vii) $-1 < x \leq 4$ (viii) $-3 \leq x < 2$

2. *Given that* $P = \{x : -3 \leq x < 4\}$ *and* $Q = \{x : -2 < x \leq 6\}$, *represent* **P n Q** *on a number line. State* **P n Q**

3. *Solve the following inequalities and represent each solution on a number line:*

(i) $5x + 7 < 3(x + 1)$ (ii) $7(2 - x) + 1 \leq 2(2x - 9)$ (iii) $5x + 3 > -11 - 2x$

(iv) $3(x - 1) + 2(x - 1) \leq 7x + 7$ (v) $\frac{3}{2} - \frac{5x}{3} > 8 + \frac{x}{2}$ (vi) $\frac{x}{4} + 3 \geq 1 + \frac{x}{2}$

(vii) $\frac{3x}{2} - \frac{2}{3}(1 - 2x) < 5$ (ix) $7 \geq 4 - 3x > -5$ (x) $2x - 4 \leq 4 > -3x - 5$

*4. Using a number line, find the integral values of **x** which satisfy the sets:*

$\{5 - 3x > -7\}$ n $\{x - 6 \leq 3x - 4\}$

*5. Find all the integral values of **x** which satisfy the inequalities:*

$\frac{5x + 7}{4} \leq \frac{3x + 5}{2} < \frac{x + 11}{3}$

*6. Find the positive integral values of **x** which satisfy the inequalities:*

$\frac{x}{4} - 3 \leq x + 2 \leq 21 - 2x$

*7. Find the greatest integral value of **x** which satisfies the inequality:*

$2 - \frac{3x}{2} > x + 3$

*8. Given that $-1 < x < 4$, find the values of **a** and **b** for which $a \leq 2x + 3 < b$*

EER:

1. Solve the inequality: $\frac{x}{4} + 5 \geq 1 + \frac{x}{2}$

2. Solve the inequality: $10x - 3(2x - 1) \geq 8x + 15$

3. Solve the inequality: $\frac{2x - 3}{5} \geq \frac{x}{2} - 1$

4. Solve the inequality: $3(x - 2) + 4 \leq 2(2x - 3)$

5. Solve the inequality: $-6 \leq 2(x - 5) < 4$

6. Solve the inequality: $-3 < \frac{3}{2}(2 - x) \leq 5$

*7. Using a number line, find the integral values of **x** which satisfy the sets:*

$\{3x > 2x + 5\}$ n $\{3x < 32 - x\}$

8. *Solve the inequality:* $\frac{1}{2} - \frac{x}{6} > -\frac{5}{2}$

9. *Find the range of values of* ***x*** *which satisfy the inequalities:*
$x - 4 \leq 3x + 2 < 2(x + 5)$

10. *Given that* $P = \{x : -4 \leq x \leq 2\}$ *and* $Q = \{x : -2 < x < 5\}$, *represent* ***P n Q*** *on a number line. State* ***P n Q***

11. *Solve the inequality:*

12. *Find all the integral values of* ***x*** *which satisfy the inequalities:*
$2x + 3 \geq 5x - 3 > -8$

13. *Find all the integral values of* ***x*** *which satisfy the inequalities:*
$2x - 4 \leq 4 > -3x - 5$

GRAPHING LINEAR INEQUALITIES

Summary:

1. *In shading out the unwanted region, we proceed as follows:*

(i) *Make* ***y*** *the subject in the given inequality equation*

(ii) *Rewrite the equation in the form* ***y = mx + c***

(iii) *Draw a solid line if the inequality is ≤ or ≥ and in case the inequality is* ***< or >,*** *draw a dotted line*

(iv) *If the inequality is* ***> or ≥ ,*** *the wanted region is above the line and If the inequality is* ***< or ≤ ,*** *the wanted region is below the line. Thus we shade out the unwanted region*

2. *The points* ***(x, y)*** *from the wanted region are called an integral solution* ***(x*** *and* ***y*** *are integers****)***

3. *The maximum and minimum values of a given expression in the wanted region will be found at one of its vertices*

EXAMPLES:

1. Given that $P = \{ (x, y) : 2x - 3y \leq 6 \}$ *and* $Q = \{(x, y) : x + y < 0\}$, *by shading the unwanted region, show the region representing **P n Q***

2. (i) By shading the unwanted region, show the region representing

$\{ (x, y) : y \geq 6 - x \text{ n } y - x > 0 \text{ n } y \leq 7 \}$

(ii) Find the integral solution of the inequalities

(iii) Calculate the area of the wanted region

*3. (i) By shading the unwanted region, show the region which satisfies the inequalities: **3x + 4y < 12, y ≥ 0** and **x ≥ 0***

(ii) Find the integral solution of the inequalities

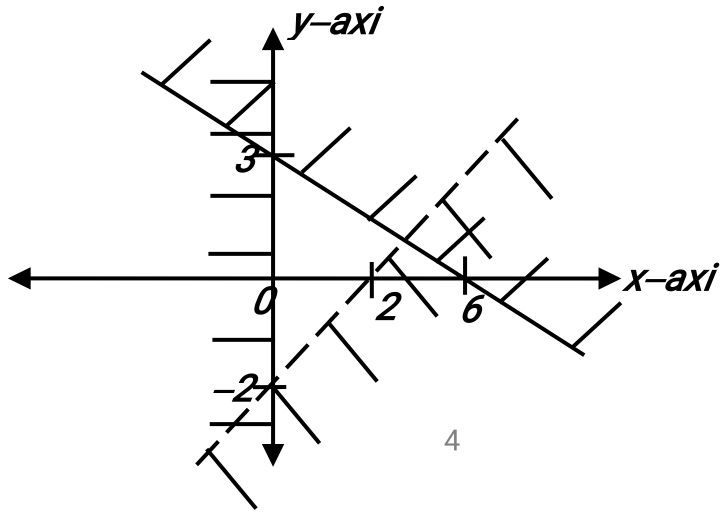
(ii) Calculate the area of the wanted region

*4. (i) By shading the unwanted regions, show clearly the region **R** which satisfies the inequalities: **y – x < 2, 2y + 5x ≤ 25** and **6y + x ≥ 5***

*(ii) Given that **P(x, y) = 50x + 40y,** determine the maximum and minimum values of **P** in the region **R.***

*(iii) Determine the area of the unshaded region **R***

5. (i) Find the inequalities satisfied by the unshaded region below:



(ii) *Calculate the area of the unshaded region*

6. *By shading the unwanted region, show the region representing* $y > x^2$ *for* $-2 \le x \le 2$

7. *By shading the unwanted region, show the region representing* $y > x^2 - 1$ *for* $-2 \le x \le 2$

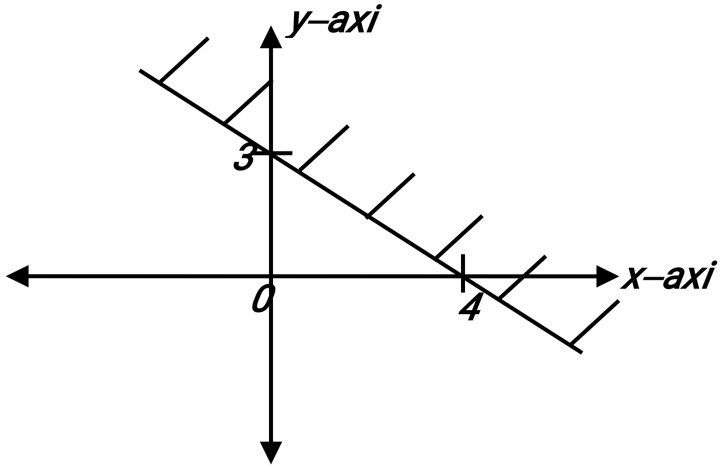
EER:

1. *By shading the unwanted region, show the region which satisfies the inequality* $3x + 4y < 12$

2. *By shading the unwanted region, show the region representing*

$\{(x, y) : y > x - 1 \text{ n } y \le 3\}$

3. *Find the inequality that satisfies the unshaded region below:*



4. (i) *By shading the unwanted region, show the region which satisfies the inequalities:* $x + y \le 3$, $y > x - 4$ *and* $y + 7x \ge -4$

(ii) *Calculate the area of the wanted region*

5. (i) *By shading the unwanted region, show the region representing*

$\{ (x, \ y) : \ y \geq x - 2 \ \ n \ \ y + x \ \leq 14 \ \ n \ \ y \ \leq 7x - 26 \ \}$

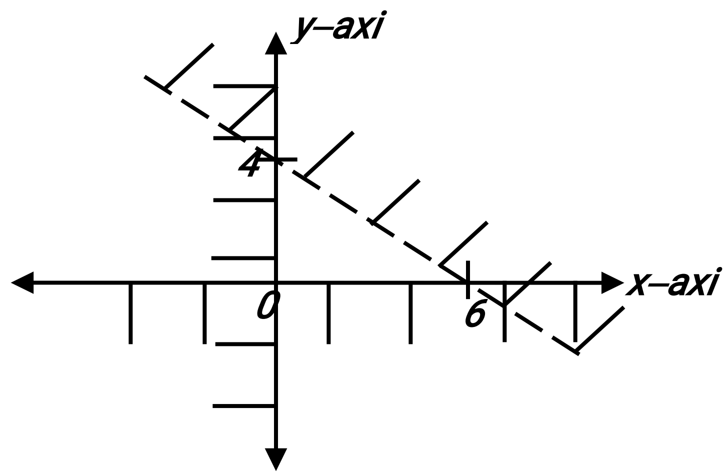
(ii) *Calculate the area of the wanted region*

6. (i) *By shading the unwanted region, show the region which satisfies the inequalities:* $x \leq 4$, $2y + x \geq 4$ *and* $4y - 3x \leq 8$

(ii) *Find the integral solution of the inequalities*

(iii) *Find the maximum and minimum values of* $P = x + y$ *in the wanted region.*

7. *Find the inequalities satisfied by the unshaded region below:*



8. *By shading the unwanted region, show the region satisfying the inequalities* $y \leq 2x + 1$ *and* $y \geq 3$

9. (i) *On the same axes, draw the curve* $y = 4 - x^2$ *for* $-2 \leq x \leq 2$ *and the line* $y = 1$

(ii) *By shading the unwanted region, show the region represented* $y \leq 4 - x^2$ *and*

$y \geq 1$

(iii) *State the integral coordinates of the points which lie in the region*

$\left\{ y \geq 1 \ n \ \ y \leq 4 - x^2 \right\}$

10. (i) *By shading the unwanted region, show the region representing*

$\{ (x,\ y) :\ y \geq 1 \ n \ \ y + x \leq 5 \ n \ \ x \geq 1 \}$

(ii) *Calculate the area of the wanted region*

QUADRATIC INEQUALITIES

Summary:

1. *Solving a quadratic inequality is the same as find the range of* ***x****–values where the graph in the equation will be above or below the x–axis*

2. *The following steps apply when solving a quadratic inequality:*

(i) *Replace the original inequality with a quadratic equation*

(ii) *Solve the equation to get the endpoints of the three different intervals*

(iii) *Plot the solution on a number line to identify the intervals for investigation*

(iv) *Pick a number from each interval and work out the sign for each interval*

(v) *The symbol in the inequality determines the required range. In any interval the graph is either above or below the x–axis*

EXAMPLES:

1. *Find the range of* ***x*** *for which* $x^2 + x - 12 \leq 0$

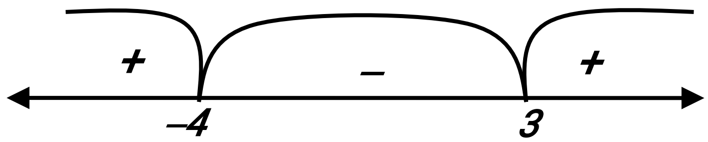
Soln:

At the endpoints, $x^2 + x - 12 = 0$

$$\Rightarrow\; x = \frac{-1 \pm \sqrt{1 + 48}}{2}$$

$\therefore \quad x = -4 \text{ or } 3$

Testing for negativity (negative sign)



Required range = $-4 \le x \le 3$

NOTE: *The final answer must have the symbol used in the original inequality*

2. *Solve for* ***x*** *in the inequality:* $x^2 - x - 6 > 0$

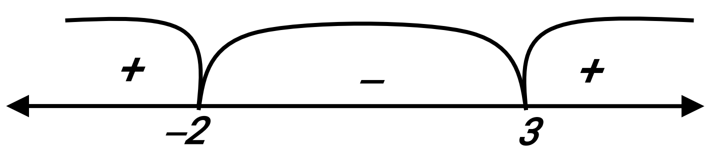
Soln:

At the endpoints, $x^2 + - x - 6 = 0$

$$\Rightarrow\; x = \frac{1 \pm \sqrt{1 + 24}}{2}$$

$\therefore \quad x = -2 \text{ or } 3$

Testing for positivity



Required range = $x < -2$ **or** $x > 3$

3. *Solve for* ***x*** *in the inequality:* $x^2 - 36 < 0$

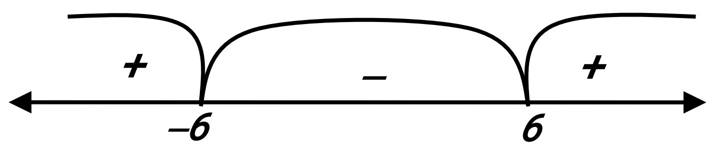
Soln:

At the endpoints, $x^2 - 36 = 0$

$\Rightarrow \; x = \pm\sqrt{36}$

$\therefore \quad x = -6 \text{ or } 6$

Testing for negativity



Required range = $-6 < x < 6$

EER:

1. *Solve for* x *in the inequality:* $x^2 - 4x + 3 < 0$

2. *Solve for* x *in the inequality:* $x^2 + 2x - 15 \geq 0$

3. *Solve for* x *in the inequality:* $(x + 2)(x - 4) < x^2 - 6$

4. *Solve for* x *in the inequality:* $2x^2 + 4x \geq x^2 + 5x + 6$

5. *Determine the solution set of the inequality:* $4x^2 - 5x - 6 < 0$

6. *Find the integral values of* x *which satisfy the inequality:*
$2x^2 + 5x - 3 < 0$