VECTORS

Summary:

1. *A vector has both magnitude and direction.*

2. $OP = \begin{pmatrix} x \\ y \end{pmatrix}$ *is the position vector of point* $P(x, \ y)$.

3. *The magnitude **or** length **or** modulus of vector **OP** is denoted by*

$|OP| = \sqrt{x^2 + y^2}$.

4. *To add two vectors we add the corresponding numbers*

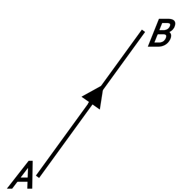
5. *To subtract two vectors we subtract the corresponding numbers*

6. *A scalar **k** multiplied by vector* $OP = \begin{pmatrix} x \\ y \end{pmatrix}$ *is treated as follows:*

$$kOP = k\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$$

7. *A displacement vector **AB** is represented by a directed line segment **AB** as*

shown:

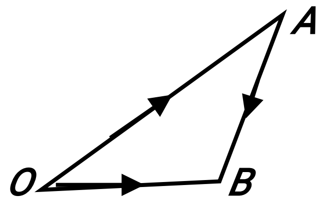


*The vectors **AB** and **BA** are equal in length but opposite in direction*

$\therefore$ ***BA = –AB***

8. *In the triangle **OAB,** the displacement **OA** followed by **AB** is equal to a single*

*displacement **OB.***



OB = OA + AB

$\therefore$ ***AB = OB – OA*** *"The vector triangle equation"*

9. *If vector **AB** is parallel to **CD,** then* $AB = kCD$

10. *If **ABCD** is a parallelogram, then the two opposite sides are parallel and also equal in length **(AB = DC** and **AD = BC).***

11. *If **AB** is parallel to **BC** with a common point **B,** then the points **A, B** and **C** are collinear (* $AB = kBC$ *)*

EXAMPLES:

1. *Given the points **A(4, 1)** and **B(12, 16),** find the:*

(i) *column vector **AB***

(ii) *length of **AB***

2. *The position vectors of **P** and **Q** are* $\begin{pmatrix} -2 \\ 13 \end{pmatrix}$ *and* $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ *respectively, find the* *magnitude of* ***PQ***

3. *Find the distance between the points* ***P(–8, 2)*** *and* ***Q(4, 7)***

4. *Given that* $OA = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ *and* $AB = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$*, find the:*

(i) *position vector of* ***B***

(ii) $|OB|$

5. *Given that* $OB = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$ *and* $AB = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$*, find the:*

(i) *coordinates of* ***A***

(ii) *modulus of* ***OA***

6. *Given the points* ***P(–2, 3)*** *and* ***Q(3, 6),*** *find the coordinates of* ***R,*** *if*

$OR = 3OP + \frac{1}{3}OQ.$

7. *Given the points* ***A(3, 4)*** *and* ***B(9, 2),*** *find the coordinates of* ***T,*** *if*

$OT = OA + \frac{1}{2}AB.$

8. *Given that* $\mathbf{a} = \begin{pmatrix} -2 \\ -9 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$ *and* $\mathbf{m} = \mathbf{a} + 2\mathbf{b}$, *find the magnitude of* ***m.***

9. *Given the vectors* $\mathbf{a} = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 8 \\ 11 \end{pmatrix}$ *and* $\mathbf{c} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$, *find the length of*

$\mathbf{a} + 2\mathbf{b} + \mathbf{c}.$

10. *Given the vectors* $AB = \begin{pmatrix} 9 \\ -7 \end{pmatrix}$ *and* $BC = \begin{pmatrix} -6 \\ 3 \end{pmatrix}$, *find the:*

(i) column vector ***AC***

(ii) modulus of ***AC***

11. *Given the vectors* $PQ = \begin{pmatrix} 13 \\ 4 \end{pmatrix}$ *and* $RQ = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$, *find:*

(i) vector ***PR***

(ii) the length of ***PR***

12. *Given the vectors* $AB = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$ *and* $AC = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, *find the magnitude of* ***BC.***

13. *If* $p = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $q = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ *and* $r = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$, *find the values of* ***a*** *and* ***b*** *such that*

$ap + bq = r.$

14. *If* $a = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $b = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ *and* $c = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$, *find the values of* ***x*** *and* ***y*** *such that*

$xa + yb = c.$

15. *If* $u = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ *and* $w = \begin{pmatrix} 8 \\ 12 \end{pmatrix}$, *find the values of* ***x*** *and* ***y*** *such that*

$xu + yv = w.$

15. ***ABCD*** *is a parallelogram with* ***A(–2, –2), B(6, –2)*** *and* ***C(2, 1).*** *Find the coordinates of* ***D.***

16. ***ABCD*** *is a parallelogram with* ***A(2, 1), B(3, 4)*** *and* ***C(–1, 2).*** *Find the*

*coordinates of **D***

17. ***PQRS** is a parallelogram with **P(1, 1), Q(5, 3)** and **R(7, 7)**. Find the:*
   (i)**column vector **PS
   ***(ii)** coordinates of **S**.*

18. ***ABCD** is a quadrilateral with **A(4, 1), B(2, –2, C(–2, 0)** and **D(0, 3)**. Show that **ABCD** is a parallelogram*

19. *The vectors* $\mathbf{a} = \begin{pmatrix} 2 \\ \lambda \end{pmatrix}$ *and* $\mathbf{b} = \begin{pmatrix} 8 \\ -12 \end{pmatrix}$ *are parallel to each other. Find the*
   value of λ

20. *The vectors* $\mathbf{p} = \begin{pmatrix} \lambda \\ 2 \end{pmatrix}$ *and* $\mathbf{q} = \begin{pmatrix} 15 \\ 6 \end{pmatrix}$ *are parallel to each other. Find the*
   value of λ

21. *Show that the points **A(–1, 3), B(2, 1)** and **C(8, –3)** are collinear.*

22. *Show that the points **P(–1, –5), Q(0, –2)** and **R(2, 4)** are collinear.*

23. *Given the points **A(–2, –1), B(1, 5)** and **C(2, 7),** find the value of **k** such that*
   $AB = kAC$, *hence state the ratio* $AB : AC$.

24. *In the vector triangle **OAB, M** is a point on **AB** such that **AM: AB = 2:5**.*
   Express:
   (i) AM** in terms of **AB
   (ii) MB** in terms of **AB
   (iii) AB** in terms of **AM
   (iv) AB** in terms of **MB
   (v) OM** in terms of **OA** and **AB

(vi) OM** in terms of **OB** and **AB

25**. In the vector triangle **OAB, K** is a point on **AB** such that **3AK = 2KB.
Express:
(i) AK** in terms of **AB
(ii) KB** in terms of **AB
(iii) AK** in terms of **KB
(iv) KB** in terms of **AK
(v) OK** in terms of **OA** and **AB
(vi) OK** in terms of **OB** and **AB

***26.** In the vector triangle **OAB, N** is the midpoint of **AB.** Express:*
(i) ON** in terms of **OA** and **AB
(ii) ON** in terms of **OB** and **AB

***27.** The position vectors of the points **A** and **B** are $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 12 \\ -11 \end{pmatrix}$*

*respectively. Point **M** is on **AB** such that **AM: AB = 2 : 3,** find the:*

(i)** column vector **AB

(ii)** column vector **AM

(iii)** position vector of **M.

***28.** Given that $OA = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, $OB = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$ and **M** is a point on **AB** such that*

***3AM = 2MB,** find the:*

(i)** coordinates of **M

(ii)** magnitude of **OM

29. *Given that* $OA = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$, $OB = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$ *and* ***M*** *is the midpoint of* ***AB,*** *find the:*

(i) *column vector* ***AB***

(ii) *position vector of* ***M***

30. *Given that* $OA = \begin{pmatrix} -1 \\ -15 \end{pmatrix}$, $OB = \begin{pmatrix} 7 \\ -11 \end{pmatrix}$ *and point* ***E*** *divides* ***AB*** *in the ratio*

1 : 3, *find the position vector of* ***E.***

31. *Given that* $OA = \mathbf{a}$, $OB = \mathbf{b}$ *and* ***M*** *is the midpoint of* ***AB,***

(a) *Draw a vector diagram showing vector* ***AB***

(b) *Express the following vectors in terms of* ***a*** *and* ***b:***

(i) AB

(ii) AM

(iii) OM

32. *In a triangle* ***OAB,*** $OA = \mathbf{a}$, $OB = \mathbf{b}$ *and point* ***K*** *divides* ***AB*** *in the ratio* ***1:2,***

Express the following vectors in terms of ***a*** *and* ***b:***

(i) AB

(ii) AK

(iii) OK

33. *In a triangle* ***OAB,*** $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ *and* ***N*** *is a point on* ***AB*** *such that*

2AB = 3NB. *Express vector* ***ON*** *in terms of* ***a*** *and* ***b.***

34. *In a triangle* ***OAB,*** $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, *point* ***C*** *divides* ***AB*** *in the ratio* ***2:3*** *and*

D *is the midpoint of* ***OC.***

(a) *Express the following vectors in terms of* ***a*** *and* ***b:***

(i) AB
(ii) OC
(iii) BD

(b) *Taking* ***O*** *as the origin, point* ***A(–15, 20)*** *and* ***B(10, 0),*** *find the:*

(i) *position vector of* ***C*** *in* ***(a)(i)*** *above.*
(ii) *coordinates of* ***C.***
(iii) *length of* ***OC.***

35. *In a triangle* ***OAB, M*** *and* ***N*** *are midpoints of* ***OA*** *and* ***OB*** *respectively.*

$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{ON} = \mathbf{b}$ *and* ***P*** *is a point on* ***AB*** *such that* ***4AP = 3AB.***

(a) *Express the following vectors in terms of* ***a*** *and* ***b:***

(i) AB
(ii) OP
(iii) MB
(iv) NP

(b) *Show that* ***AB*** *is parallel to* ***MN.***

36. *In a triangle* ***OAB, M*** *and* ***N*** *are midpoints of* ***AB*** *and* ***OB*** *respectively.*

$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{ON} = \mathbf{b}$ *and* ***P*** *is a point on* ***OM*** *such that* ***3OP = 2OM.***

(a) *Express the following vectors in terms of* ***a*** *and* ***b:***

(i) AB
(ii) OM
(iii) PB
(iv) AP

(b) (i) *Show that the points* ***A, P*** *and* ***N*** *are collinear.*
(ii) *Find the ratio in which* ***P*** *divides* ***AN.***

37. *In a triangle* ***OAB,*** $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, ***P*** *and* ***Q*** *are points on* ***OA*** *and* ***AB*** *respectively such that* ***3OP = PA, AQ = 2QB*** *and* ***N*** *is the midpoint of* ***OQ. ANM*** *is a straight line which is such that* ***AN = 5NM.*** *Given also that* $\overrightarrow{OM} = h\,\overrightarrow{OB}$, *where* ***h*** *is a scalar.*

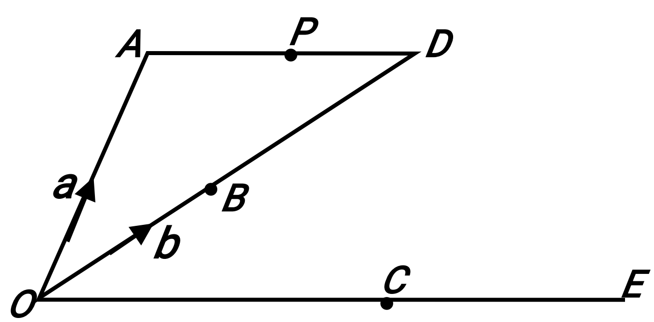
(a) *Express the following vectors in terms of* ***a*** *and* ***b:***

(i) OQ
(ii) AN
(iii) PN
(iv) NB

(b) *Show that the points* ***P, N*** *and* ***B*** *are collinear*

(c) *Find the value of* ***h.***

38. *In the figure below,* ***P*** *is a point on* ***AD*** *such that* ***PD : AP = 1 : 2,*** $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, ***3OB = 2BD*** *and* ***OC = 3CE = 3AP.***



(a) *Express the following vectors in terms of* ***a*** *and* ***b:***

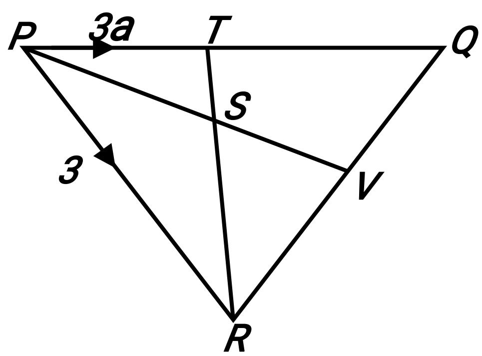
(i) AD

(ii) BP

(iii) DC

(b) *Show that* ***AD : OE = 3 : 8***

39. *In the figure below,* $PT = 3a$, $PR = 3b$, $PQ = 4PT$, $2PS = PV$ *and* $3RS = 2RT$.



(a) *Express the following vectors in terms of* ***a*** *and* ***b*****:**

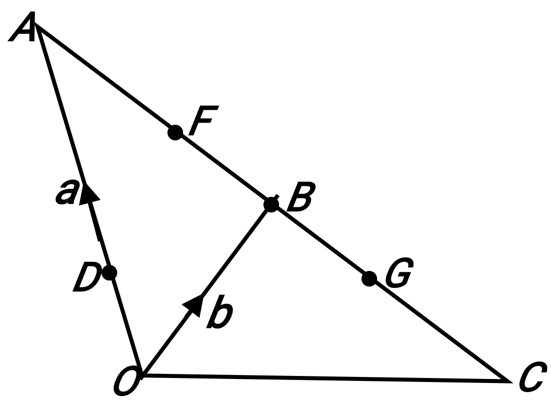
(i) RS

(ii) PV

(iii) RQ

(b) *Find the ratio of* ***RV*** *to* ***RQ*****.**

37. *In the figure below,* $OA = a$, $OB = b$, ***F*** *and* ***G*** *are points on* ***AC*** *such that* ***AF : AB = 3 : 4*** *and* ***AG : AC = 2 : 3.*** *Point* ***D*** *is on* ***OA*** *such that* ***OD : DA = FB : BG = 1 : 2.***



(a) *Express* ***AG*** *and* ***AC*** *in terms of* ***AB.*** *Hence find the following vectors in*

terms of ***a*** *and* ***b*****:**

(i) AB

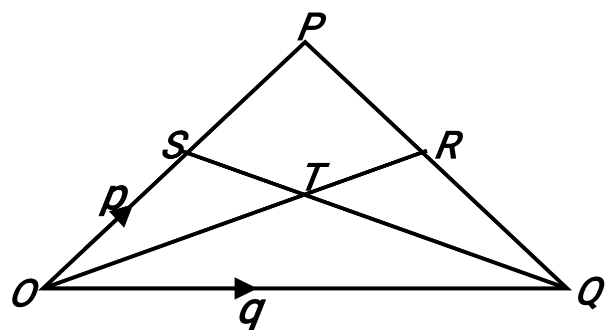
(ii) AC

(iii) DG

(iv) OF

(b) *Find the ratio* ***DG : OC***

39. *In the figure below,* $OP = p$, $OQ = q$, $OS = \frac{3}{4}OP$ *and* ***PR : RQ = 2 : 1***



(a) *Express the following vectors in terms of* ***p*** *and* ***q:***

(i) PQ

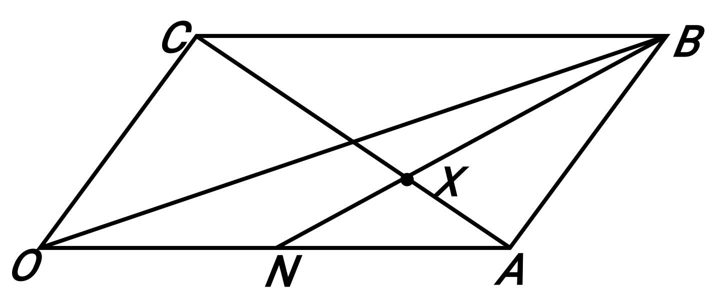
(ii) OR

(iii) SQ

(b) *Line* ***OR*** *and* ***SQ*** *meet at point* ***T*** *such that* ***OT = hOR*** *and* ***ST = kSQ.***
(i) *By expressing* ***OT*** *in two different ways, find the values of* ***h*** *and* ***k***
(ii) *Determine the ratio in which* ***T*** *divides* ***SQ***

40. *In the figure below,* ***OABC*** *is a parallelogram where* ***OA = a*** *and*

AB = b. *Point* ***N*** *is on* ***OA*** *such that* ***ON:NA = 1:2.***



(a) *Express the following vectors in terms of* ***a*** *and* ***b:***

(i) ***AC***

(ii) ***BN***

(b) *Line* ***AC*** *and* ***BN*** *meet at point* ***X*** *such that* ***AX = hAC*** *and* ***BX = kBN***

(i) *By expressing* ***OX*** *in two different ways, find the values of* ***h*** *and* ***k***

(ii) *Determine the ratio in which* ***X*** *divides* ***AC***

35. *In a triangle* ***OAB,*** $OA = a$, $OB = b$, ***N*** *and* ***M*** *are points of* ***AB*** *and* ***OB*** *respectively. Line* ***ON*** *and* ***AM*** *meet at point* ***T*** *such that* ***AT = TM*** *and* $OT = \frac{3}{4}ON$. Given that ***OM = xOB*** and ***AN = yAB,*** *Express the vectors:*

(i) ***AM*** *and* ***OT*** *in terms of* ***a, b*** *and* ***x.***

(ii) ***ON*** *and* ***OT*** *in terms of* ***a, b*** *and* ***y,*** *hence find the values of* ***x*** *and* ***y.***

EER:

1. *Given the points* ***A(3, 4)*** *and* ***B(9, 1),*** *find the coordinates of* ***P,*** *if*

$OP = OA + \frac{1}{3}AB.$

2. *Given the vectors* $a = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ m \end{pmatrix}$ *and* $c = \begin{pmatrix} n \\ -2 \end{pmatrix}$, *find the values of* ***m*** *and* ***n*** *for which* $4a + 2b = 3c.$

3. *Given the vectors* $p = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $q = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ *and* $r = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$, *find the values of* ***a*** *and* ***b*** *for which* $ap + bq = r.$

4. Given that vector $OA = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $OB = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$, find the magnitude of vector $P = OA + \frac{2}{3}AB$.

5. Given that vector $p = \begin{pmatrix} x \\ x+2 \end{pmatrix}$, find the possible values of ***x*** for which $|p| = 10$.

6. The position vectors of the points ***A*** and ***B*** are $\begin{pmatrix} -1 \\ -15 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ -11 \end{pmatrix}$ respectively. If point ***E*** divides ***AB*** in the ratio ***1 : 3***, find the position vector of ***E***.

7. Find the distance between the points ***P(–8, 2)*** and ***Q(4, 7)***

8. Given the points ***A(–1, 2), B(2, 8), C(–2, –5)*** and ***D(4, y),*** find the value of ***y*** for which ***AB*** is parallel to ***CD.***

9. Given the vectors $p = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $q = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $r = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$, find the values of ***a*** and ***b*** for which $ap + bq = r$.

1. Given the vectors $AB = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$ and $CB = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$, find the:

(i) vector ***AC***

(ii) magnitude of ***AC***

13*. The vectors* $OP = \begin{pmatrix} -1 \\ -15 \end{pmatrix}$, $OQ = \begin{pmatrix} 7 \\ -11 \end{pmatrix}$ *and* $PN = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$.

(a) *Find the:*

(i) *position vector of* ***N*** ***(02 marks)***

(ii) *length of* ***ON*** ***(02 marks)***

(iii) *coordinates of point* ***E,*** *where* ***E*** *divides* ***PQ*** *in the ratio* ***1:3.*** ***(03 marks)***

(b) *Use the vector method to show that* ***N*** *lies on* ***PQ.*** *Hence state the ratio* ***PN : PQ.*** ***(05 marks)***

5. *Given that* ***OA = a, OB = b*** *and* ***C*** *is the midpoint of* ***AB,***

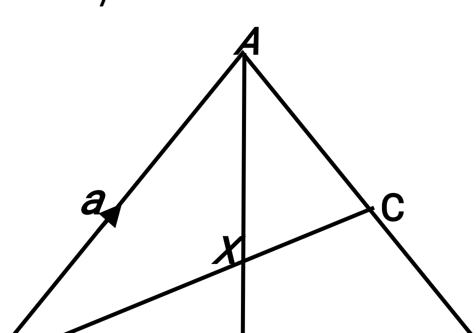
(a) *Draw a vector diagram showing vector* ***AB.*** ***(01 mark)***

(b)*Express in terms of* ***a*** *and* ***b*** *the vectors:*

(i) AB ***(01 mark)***

(ii) OC ***(02 marks)***

37. *In the triangle* ***OAB,*** $OA = a$, $OB = b$, ***C*** *is a point on* ***AB*** *such that* ***AC:AB = 1:3*** *and* ***D*** *is the midpoint of* ***OB.***



(a) *Express the following vectors in terms of* ***a*** *and* ***b:***

(i) AB
(ii) OC
(iii) AD

(b) X *is a point on* ***AD*** *such that* ***AX : AD = 4 : 5.*** *Find in terms of* ***a*** *and* ***b*** *the vectors:*

(i) AX
(ii) OX

(c) *Find in simplest form the ratio* ***OX : OC.***

TRANSLATION

Summary:

1. *Translation deals with movement of an object to a new position*

2. *A translation* $T = \begin{pmatrix} a \\ b \end{pmatrix}$, *means that an object is moved a distance* ***a*** *in the* ***x****–direction and a distance* ***b*** *in the* ***y****–direction*

3. *A translation* $T = \begin{pmatrix} a \\ b \end{pmatrix}$, *moves point* ***P(x, y)*** *to a new position*

$P^{I}(x + a,\ y + b)$. *Thus* , ***Translation + object = image***

EXAMPLES:

1. *A translation* $T = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, *maps the points* ***P(3, 7)*** *and* ***Q(6, 1)*** *onto the points* P^{I} *and* Q^{I} *respectively. Find the coordinates of* P^{I} *and* Q^{I}

2. *A triangle with vertices* ***A(2, 1) B(2, 3)*** *and* ***C(4, 1)*** *is mapped onto its image by a translation* $T = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. *Find the coordinates of the image of the triangle* ***ABC.***

3. *A translation* $T = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ *maps point* ***P*** *onto* $P^{I}(-1, 4)$. *Find the coordinates of point* ***P***

4. *Find the translation that maps point A****(2, 6)*** *onto* $A^{I}(3,\ 8)$.

5. *A translation* ***T*** *maps point* ***P(2, 5)*** *onto* $P^{I}(3,\ 2)$. *Find the image of* ***Q(5, 7)*** *under translation* ***T***

6. *A triangle with vertices* ***A(1, 2) B(3, 4)*** *and* ***C(5, 2)*** *is mapped onto its image by a translation* $T_1 = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ *followed by a translation* $T_2 = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

Find: ***(i)*** *a single translation representing the two successive translations*

(ii) *the coordinates of the image of the triangle* ***ABC.***

***7.** A triangle with vertices **A(2, 0) B(1, –3)** and **C(–2, 1)** under goes a translation* $T_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ *to give triangle* $A^I B^I C^I$. *Triangle* $A^I B^I C^I$ *is then mapped onto triangle* $A^{II}\ B^{II}\ C^{II}$ *by a translation* $T_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

***(a)** Find the coordinates of the vertices of:*

***(i)** triangle* $A^I B^I C^I$

***(ii)** triangle* $A^{II}\ B^{II}\ C^{II}$

***(b)** Plot triangle **ABC** and its images on the same axes.*

***8.** A translation* $T = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, *maps the line **y = 2x + 1** onto its image. Find the equation of the image line*