SENIOR SIX PHYSICS PAPER TWO (WAVES), CONTINUATION…..
(*all the streams should copy*) *BY MR.ALIA AMBROSE*

MUSICAL NOTES AND SOUND

SOUND

Sound is any mechanical vibration whose frequency lies within the audible range. sound waves do not pass through the vacuum. It requires a material medium for their transmission.

Experiment to show that sound waves requires a material medium for transmission.
(leave about 6 lines for the diagram)

When all the air has been removed (pumped out) thus creating a vacuum in the bell jar, no sound is heard although the harmer is seen hitting the gong. This shows that sound requires a material medium for their transmission.

Compression and rarefaction

When sound waves pass through air, variation in pressure occurs. somewhere it is below the normal and at some other point it is above the normal. The condition of air when the pressure is above the normal is called compression and the condition when the pressure is below the normal is called rarefaction.

Qn. Sound propagation in air is considered to be an adiabatic process. Explain.

Compression of air causes a rise in temperature unless heat is withdrawn. Rarefaction causes a decrease in temperature unless heat is added. Because air is a poor conductor of heat, no much heat enters a region of rarefaction nor leaves a region of compression hence sound propagation is an adiabatic process. No heat leaves or enter the system.

Characteristics of sound waves.

They undergo reflection, refraction, diffraction and interference.
Diffraction.is the spreading of sound waves when they pass through openings or around obstacles. Diffraction of sound waves depends on;
(i) The size of the aperture; The smaller the aperture the greater the diffraction.
(ii) The wavelength; Diffraction increases with wavelength. Diffraction explains why light travels in a straight line and why we are able to hear around corners.
Light waves has a very short wavelength and therefore no appreciable diffraction is obtained. Sound waves have long wavelength therefore diffraction can be observed thus able to hear around corners.

QN; Explain why we are able to hear sound easily at night than during day.
At night, layers of air near the earth are colder than those high up and hence sound waves are now refracted towards the earth with consequent increase in intensity.
Reflection. Sound waves can be reflected by obstacles obeying the laws of reflection. A reflected sound from an obstacle is known as an echo. The time that elapses between the original sound and the echo determines whether the observer will be able to distinguish the original sound from echo. The time depends on;

(i) The distance travelled by the echo from the reflection or obstacle.
(ii) Speed of sound.
If the time is less than 0.1 seconds, the human ear cannot distinguish between the original sound and the echo. If the time is about 0.1 seconds, the original sound is heard prolonged to the listener. This effect is called **reverberation.**

Explanation of reverberation

When sound is reflected from a hard surface or obstacle close to the observer, an echo follows the original incident sound so closely that the listener cannot distinguish between the original sound and echo. The observer however gets an impression of a prolonged original sound. This effect is what is known as **reverberation.** The time taken for the intensity of sound to completely die out is called **reverberation time.**

Implications of reverberation.

(i) In large halls, soft clothings, cushions and human skin absorb sound reducing its intensity. This may cause the music or speech to become weaker and inaudible. In such cases reverberation can improve audibility of sound.
(ii) Excessive reverberation however makes the speech or music to sound indistinct and confused.

MUSICAL NOTES

A musical note or tone is a sound of regular frequency. Music is a combination of such sound.
Noise is sound produced by sources vibrating with no fixed frequency (irregular frequency).

Characteristics of musical notes (musical sound)

A musical note (sound) has three major characteristics and these are;
- pitch
- Quality (timbre)
– Loudness (intensity)

(i) **Pitch**. This is the characteristic of a note which enables one to differentiate a high note from a low note. It depends on frequency of sound. The higher the frequency, the higher the pitch.

(ii) **Quality or timbre;** A musical note played on a piano sound different from the one played on a drum although the same note may be of the same pitch. This characteristic that distinguishes one note from the other of the same frequency (pitch) is what we called the **quality (timbre).** of a musical note.

(iii) **Loudness (intensity).** This is the energy crossing a unit area around a point in one second. Loudness increases with intensity which is proportional to the square of the amplitude of the wave (sounding body). $I \alpha a^2$. where I is the intensity and a is the amplitude. The greater the intensity of sound, the louder the sound. Also, the intensity of sound decreases as the distance from the source increases according to the inverse square law. This is because of two major reasons.
– As sound travels further away from the source, some energy is lost to the transmitting medium.
– As sound travels further away from the source, it’s energy spreads over a wider area .The intensity at any given point which is at a distance r from the source is

given by $I = \frac{P}{A}$;Where P is power (rate at which energy is transfered by the wave) and A is the area.

I = $\frac{P}{4\pi r^2}$ = $(\frac{P}{4\pi})\frac{1}{r^2}$ but $\frac{P}{4\pi} = constant$. This implies that I $\propto \frac{1}{r^2}$. it can now be seen that intensity of sound is inversely propertional to the square of distance from the source. This explains why intensity decreases wiyh increasing distance from the source

QN. Explain why the amplitude of a wave decreases as the distance from the source increases.

Soln;

As the distance from the source increases, there is a decrease in intensity of sound waves caused by;

(i) Loss of energy of the wave to the transmitting medium.

(ii) The wave energy spreading over a wider area of a point at a distance d from the source. I $\propto \frac{1}{d^2}$. But I $\propto a^2$. This implies $a^2 \propto \frac{1}{d^2}$. Hence $a \propto \frac{1}{d}$. Therefore, the amplitude is inversely proportional to the distance of the wave from the source and so decreases as the distance increases.

QN. In what ways do musical notes different from one another (06 marks). (answer; explain the characteristics of musical notes).

VELOCITY OF SOUND

The velocity of sound waves through any material depends on;

(i) It's density,ρ

(ii) It's modulus of elasticity, E.

Thus, if V is the velocity, then $V = KE^x\rho^y$ where K is a constant and x and y are indices found by method of dimentions. Now $[V] = LT^{-1}$, $[\rho] = ML^{-3}$, $[E] = ML^{-1}T^{-2}$

$LT^{-1} = (ML^{-1}T^{-2})^x(ML^{-3})^y$ this implies $LT^{-1} = M^{(x+y)}L^{(-x-3y)}T^{-2x}$

Equating the powers of M, L and T on both sides

0 = x + y, implying x = -y, 1 = -x – 3y substituting for y we get 1 = -x – 3(-x)

X = $\frac{1}{2}$ and y = $\frac{-1}{2}$

From V = K$E^x\rho^y$, V = $E^{\frac{1}{2}}\rho^{\frac{-1}{2}}$ since K =1

Therefore $V = \sqrt{\frac{E}{\rho}}$.

In solids E is young's modulus, in case of fluids (air, gases and liquid); E is replaced by bulk modulus. Bulk modulus is defined as

$E = \frac{bulk\ stress}{bulk\ strain}$. Where bulk stress = increase in force per unit area and bulk strain $=\frac{change\ in\ volume}{original\ volume}$;therefore ;$E = \frac{\Delta P}{-\Delta V/_{V}}$,ΔP Increase in pressure ($\frac{F}{A}$) , $-\Delta V$ Change in volume (because increase in pressure causes a decrease in volume)

E = $\frac{-V\Delta P}{\Delta V}$, as $\Delta V \to 0\ and\ \Delta P \to 0, then\ \frac{\Delta P}{\Delta V} \to \frac{dP}{dV}$ Implying that E = $\frac{-VdP}{dV}$ ....(i)

But sound propagation in air is an adiabatic process. For adiabatic process $PV^{\gamma} = constant$.where $\gamma = \frac{C_p}{C_v}$ ratio of molar heat heat capacities of a gas.

Differentiating with respect to V gives $\gamma p = -V\frac{dP}{dV}$ so from (i), E = γp implying that

$V = \sqrt{\frac{\gamma p}{\rho}}$.

This makes it to appear as if the speed of sound in a gas depends on the pressure .However from $PV = nRT$ .We have $PV = \frac{m}{M}RT$ Implying $p = \frac{\rho}{M}RT$ Since R,M and T are constant,it implies that density of a gas is proportional to pressure and therefore $\frac{P}{\rho}$ is a constant.Thurs the speed of sound in a gas is independent of the pressure of a gas .suppose that a mole of a gas has a mass M and volume Then $\rho = \frac{M}{V}$ such that $V = \sqrt{\frac{\gamma p}{\rho}}$ becomes $V = \sqrt{\frac{\gamma PV}{M}}$ and since for n mole of a gas $PV = nRT$ we have $V = \sqrt{\frac{\gamma nRT}{M}}$,for one mole of a gas n = 1

$V = \sqrt{\frac{\gamma RT}{M}}$ Where R = the universal molar constant (J$K^{-1}mol^{-1}$), n = the number of moles of a gas in mass M (mol) , T = temperature of a gas in kelvin, M = mass per mole of the gas (kgmol^{-1}). Since R,γ and M are constant, it follows that the velocity of sound is proportional to the square root of its absolute temperature , thus $V\alpha\sqrt{T}$ Implying $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$

EXAMPLES

1. (a) The velocity of sound in air at 16^0C is 340ms^{-1} .Find it's velocity at 0^0C.
   (b) the velocity of sound in air at 14^0C is 340ms^{-1}.What will it be when the pressure of the gas is doubled and it's temperature raised to 157.5^0C.
   soln; T_1 = 298K , V_1 = 340ms^{-1}, T_2 = 273K, V_2 = ? . Substituting into $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$ It gives V_2 = 349.8ms^{-1}
   (b) change in pressure has no effect on the velocity of sound. T_1 = (273+14)K , V_1 = 340ms^{-1} , T_2 = (273+157.5)K , V_2 = ? Substituting into $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$ gives
   V2 = 416.4ms^{-1}
2. The wavelength of the note emitted by a tuning fork of frequency 512Hz in air at 17^0C is 66.5cm. if the density of air at s.t.p is 1.293kgm^{-3} .Calculate the ratio of molar heat capacity of air (density of mercury is 13600kgm^{-3}) .
   soln from V = λf, velocity of sound at 17^0C ,V= 512x0.665 =340.48ms^{-1}.

Now ,V = 340.48ms-1 , T= (273+17)K , V_0 = ? ,T_0 = 273, Where V_0 is the velocity at 0^0C. Substituting into $\frac{V_0}{V} = \sqrt{\frac{T_0}{T}}$ gives V_0 = 330.35ms^{-1} .

At V_0, $p = 0.76m\ of\ mercury = (0.76\ X\ 13600\ X\ 9.81)Nm^{-2}$,ρ =1.293kgm^{-3} from $V^0 = \sqrt{\frac{\gamma p}{\rho}}$, $\gamma = 1.4$

3. The velocity of sound in air at 16^0C is 340ms^{-1}. Find the wavelength os sound waves in air of note of frequency 680Hz at 16^0C and 51^0C .

(b) Calculate the velocity of sound in air at 100^0C if the density of air at s.t.p is 1.29kgm^{-3},density of mercury at 0^0C is 13600kgm^{-1},the specific heat capacity of air at constant pressure is 1.02$kJkg^{-1}K^{-1}$ and the specific heat capacity at constant volume is 0.72$kJkg^{-1}K^{-1}$.

Soln; $\lambda_{16^0c} = \frac{340}{680} = 0.2m$ .Then using $\frac{V_2}{340} = \sqrt{\frac{273+51}{273+16}}$ gives V_2 = 360ms^{-1} this then implies that $\lambda_{51^0c} = \frac{360}{680} = 0.53m$

(b) p=(0.76 x 13600 x 9.81)Nm^{-2} , $\gamma = \frac{1.02}{0.72}$, $\rho = 1.293kgm^{-3}$ so from $V_0 = \sqrt{\frac{\gamma p}{\rho}}$, V_0 = 333.3ms^{-1} Now using $\frac{V_{100}}{V_0} = \sqrt{\frac{373}{273}}$, $V_{100} = 388ms^{-1}$

Harmonics and overtones

No instrument can produce a pure note (a note of a single frequency). Notes obtained from any vibrating instrument are made of several frequency blended together. The strongest audible frequency present is the fundamental frequency and the corresponding note is the fundamental note.

Definitions

1 Fundamental frequency (first harmonic). Is the lowest frequency that a vibrating instrument can produce.

2. Harmonics; is one of the frequencies that can be produced by a particular instrument and it is a multiple of a fundamental frequency.

3. The n^{th} harmonic; is a note whose frequency is n times that of the fundamental frequency where n is a whole number

4. Overtones. These are notes of higher frequencies which are produced with the fundamental frequency.

WAVES IN PIPES

When air is blown in a pipe, a longitudinal wave is formed. This wave travels along the pipe and if the pipe is closed the waves will be reflected back. Since the incident and reflected waves have the same speed, frequency and amplitude, a stationary wave results.

Resonance of air in pipes

If a sounding /vibrating tuning fork is held over the open end of a pipe, it acts as an external periodic frequency that forces air inside the pipe to vibrate. If the frequency of the tuning fork equals to the external frequency of the air in the pipe, the air vibrates with

maximum amplitude and resonance occurs.
At resonance, a loud sound is heard by the observer. At this point, A stationary wave will have been obtained in the pipe.
The first position of resonance corresponds to the formation of first harmonic (fundamental mode of vibration). Therefore, the first loud sound is always heard when the sound is producing first harmonic. The frequency of the tuning fork at this point equals to the fundamental frequency of the pipe. By increasing the length of the pipe, Other positions of resonance will be obtained corresponding to overtones of the pipe.
NOTE; When a loud sound is heard, the frequency of the sound equals to the frequency of the tuning fork used.
Definition; Resonance is said to occur when a body is set into vibration at its own natural frequency by another body which is vibrating at the same frequency usually producing a larger amplitude of vibration. **OR;** Resonance is said to have occurred if a body vibrates at its natural frequency due to impulse received from a nearby source oscillating at the same frequency.

Explanation of fundamental note and overtone;

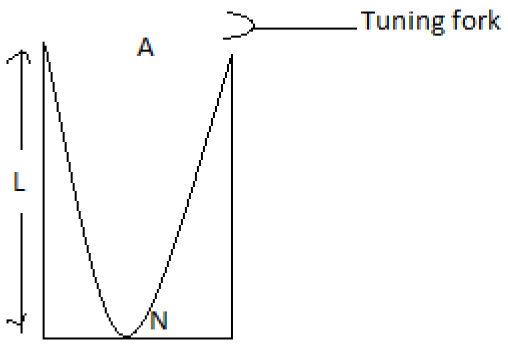
When air is blown into a pipe, it vibrates in many different modes, producing notes of different frequencies. The note of lowest frequency is the dominant one and is called fundamental note. The other notes have frequencies higher than that of the fundamental not and usually less intense. They are called overtones.

There are two types of pipes for air vibrations and these are
(i) Closed pipes; is the one in which one end is open ,while the other is closed. Eg a long drum
(ii) Open pipes; is the one that has both ends open. Eg ; a flute, a trumpet etc

(a) Modes of vibration in closed pipes

For closed pipes, a node is formed at a closed end and an antinode at the open end.

First overtone / fundamental note



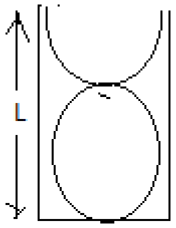
The length of the air column is L

L = $\frac{\lambda}{4}$ implying $\lambda = 4L$. So from $V = \lambda f_0$,$f_0 = \frac{V}{4L}$,f_0 = is the fundamental frequency.

QN . Describe the motion of air in a tube closed at one end and vibrating in it's fundamental frequency.

Sonl; Air at end A vibrates with maximum amplitude. The amplitude of vibration decreases as the end N is approached. Air is stationary. N is a node and A is antinode.

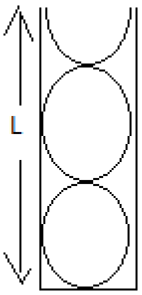
First overtone / second harmonic



The length of the air column is L

L = $\frac{3}{4}\lambda$ Implying $\lambda = \frac{4L}{3}$ But V = λf_1 therefore $f_1 = \frac{3V}{4L} = 3f_0$

Second overtone /third harmonic



L is the length of the air column

L = $\frac{5}{4}\lambda$ implying $\lambda = \frac{4}{5}$L ; from V = λf_2 .so $f_2 = \frac{5V}{4L} = 5f_0$

In closed pipes, only odd harmonics are produced ie f_0, $3f_0$, $5f_0$, $7f_0$ ....

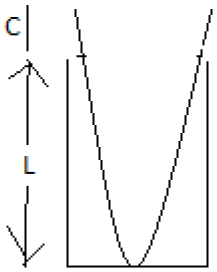
For a closed pipe, f = $\frac{nV}{4L}$ $, n = 1,3,5,7,9, \ldots \ldots \ldots \ldots \ldots$ n^{th} harmonic

Variation of pressure with displacement of air in a closed pipe

At the mouth of the pipe, the air is free to move and therefore the displacement of air molecules is large and pressure is low. At the closed end the air is less free and the displacement is minimal and the pressure is high.

END CORRECTION.

The antinode of a stationary wave in a pipe is not formed exactly at the end of the pipe. Instead it is displaced by a distance C beyond the end of the pipe. This distance is called the end correction. The effective length of a wave in a closed pipe of length, L is L+C



L+C = $\frac{\lambda}{4}$

The effective length in an open pipe of length L is L+2C

NOTE The end correction, c is related to the radius of the pipe by an equation C = 0.6r implying that the end correction is more significant for wider pipes.

Pitch and end correction

For a closed pipe, the fundamental frequency $f_0 = \frac{V}{4(L+C)}$. When the length of the pipe or tube is kept constant, f increases as c decreases. Therefore, pitch which depends on frequency is affected by cand thurs tube of smaller diameter (end correction, c) produces higher pitch than the one of larger diameter (end correction) though they will be of the same length.

Effect of temperature on the end correction

From $f_0 = \frac{V}{4(L+C)}$, also the velocity of sound, V in air at θ^0C is related to its velocity V_0 at 0^0C by $\frac{V}{V_0} = \sqrt{\frac{273+\theta}{273}} = \sqrt{1 + \frac{\theta}{273}}$ since $V \propto \sqrt{T}$

Implying V= $(\sqrt{1 + \frac{\theta}{273}})V_0$ substituting for V, $f_0 = \frac{V_0}{4(L+C)}\sqrt{(1 + \frac{\theta}{273})}$

If follows that for a given pipe, the fundamental frequency increases as temperature increases.

EXAMPLES.

1. A cylindrical pipe of length 30cm is closed at one end. The air in the pipe resonates with a tuning fork of frequency 825Hz sounded near the open end of the pipe. Determine the mode of vibration of air assuming there is no end correction. Take the speed of sound in air as $330ms^{-1}$

Soln. $F = \frac{nV}{4L}$ implying $825 = \frac{n \, x \, 330}{4 \, x \, 0.3}$, n = 3. But n = 1, 3, 5, 7, ... the mode of vibration is third harmonic

2. A cylindrical pipe of length 29cm is closed at one end. The air in the pipe resonates with the tuning fork of frequency 860Hz sounded near the open end of the pipe.

Determine the mode of vibration of air and end correction. Take the speed of sound in air as $330ms^{-1}$

Soln;

$f_n = \frac{nV}{4L}$, $860 = \frac{nx330}{4x0.29}$,n = 3.02 but for closed pipes, n = 1, 3, 5, 7, ... so the mode of vibration is fifth harmonic

$f_n = \frac{nV}{4(L+C)}$, $860 = \frac{5x330}{4(0.29+c)}$, C = 0.1897m

3. A long tube partially immersed in water and a tuning fork of 425Hz is sounded and held above it. If the tube is gradually raised, find the length of the air column when resonance first occurs. (speed of sound in air is $340ms^{-1}$)

From $v = \lambda f$, $\lambda = \frac{340}{425}$, for fundamental frequency (first loud sound) $L = \frac{\lambda}{4}$,

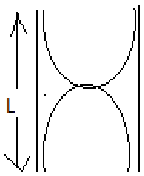
so $L = \frac{0.8}{4}$ =0.2

OPEN PIPES.

In open pipes, antinodes are found at the two open ends of the pipe.

Modes of vibration in open pipes.

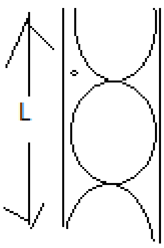
First harmonic / fundamental note.



$L = \frac{\lambda}{2}$, implying $\lambda = 2L$

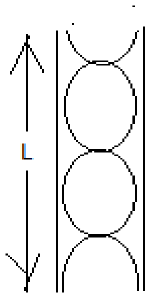
From $V = \lambda f_0$, $f_0 = \frac{v}{2L}$. F_0 is the fundamental frequency

Second harmonic / first overtone.



$L = \lambda$, from $V = \lambda f_1$ implying $f1 = \frac{V}{L} = 2\left(\frac{V}{2L}\right) = 2f_0$

Third harmonic / second overtone.



$L = \frac{3}{2}\lambda$, $\lambda = \frac{2L}{3}$, from $V = \lambda f_2$, $f_2 = \frac{3V}{2L}$, implies $f_2 = 3f_0$

NOTE; In open pipes all harmonics are present, that is f_0, $2f_0$, $3f_0$, $4f_0$, $5f_0$, ...

Open pipes produce both even and odd harmonics and this is why open pipes are preferred as musical instruments.

Generally; $f_n = \frac{nV}{2L}$,where n = 1,2,3,4,5, ... n^{th} harmonic.

EXAMPLES

The frequency of the third harmonic in an open pipe is 660 Hz, if the speed of sound in air is $330ms^{-1}$.Find

(i) The length of the air column
(ii) The fundamental frequency.

Soln; from $V = \lambda f$, $\lambda = \frac{330}{660} = 0.5m$

For third harmonic, $L = \frac{3}{2}\lambda$, implying $L = \frac{2}{3}x0.5 = 0.75m$

(ii) $F_2 = 3f_0$, $f_0 = \frac{660}{3} = 220Hz$

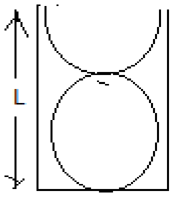
2. If the velocity of sound in air is $330ms^{-1}$ and the fundamental frequency is 110Hz in a closed pipe

(i) what is the approximate length of the tube is air resonate at the first overtone

(ii) the tube was open at both ends

Soln; (i) $f_0 = 110Hz$,$V = 330ms^{-1}$

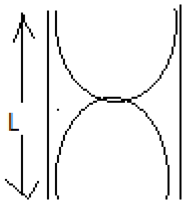
For second overtone,



$f_1 = 3f_0$, $f_1 = 3 \times 110 = 330ms^{-1}$

$V = \lambda f_1$, $\lambda = \frac{V}{f_1} = \frac{330}{330} = 1m$, $L = \frac{3}{4}\lambda = \frac{3}{4}x1 = 0.75m$

ii. for open pipe



$L = \frac{\lambda}{2}$, $\lambda = 2 \times 0.75$, $\lambda = 1.5m$

From $V = \lambda f_0$, $f_0 = \frac{330}{1.5}$, $f_0 = 220Hz$

3). Two organ pipes of length 50cm and 51cm respectively give beats of frequency 7Hz when sounding their fundamental notes together. Neglecting the end correction, calculate the velocity of sound in air.

Soln;

$$f_{0_1} = \frac{v}{2l_1} \ and \ f_{0_2} = \frac{v}{2l_2}$$

$f_b = \frac{v}{2l_1} - \frac{v}{2l_2} = \frac{v}{2}(\frac{1}{l_1} - \frac{1}{l_2})$, $7 = \frac{v}{2}(\frac{1}{0.5} - \frac{1}{0.51})$ implying that $v = 357ms^{-1}$

4. Two open pipes of length 50cm and 50.5cm produce 3 beats per seconds. Calculate the velocity of sound in air.

(b) Neglecting the end correction find the length of

(i) A closed pipe

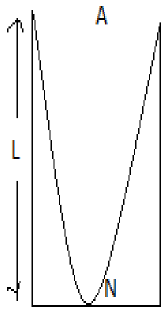
(ii) An open pipe, each of which emits a fundamental note of frequency 250Hz (velocity of sound in air $=330ms^{-1}$)

Soln; (a) $f_0 = \frac{V}{2L}$ for an open pipe.

$f_{0_1} = \frac{V}{2x50} = \frac{V}{100}$, also $f_{0_2} = \frac{V}{2x50.5} = \frac{V}{101}$, beat frequency, f =3Hz

$f = f_{0_1} - f_{0_2}$, $3 = \frac{V}{100} - \frac{V}{101}$, $V = 30300cms^{-1}$ or $V = 330ms^{-1}$

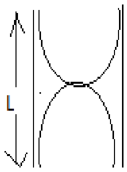
For closed pipes, $f_0 = 256Hz$.



$f_0 = \frac{V}{4L}$

implying $L = \frac{V}{4f_0}$ $L = \frac{330}{4X256} = 0.322m$

ii. for open pipes,



$f_0 = \frac{V}{\lambda} = \frac{V}{2L}$, $L = \frac{V}{2f_0} = \frac{330}{2x256} = 0.644m$

5. Calculate the frequency of the third harmonic of a sound note set up in a pipe of length 0.5m when the pipe is
   (i) Open at both ends
   (ii) Closed at one end (assume speed of sound in air = $340ms^{-1}$

Soln;

for third harmonic; $L = \frac{3}{2}\lambda \Rightarrow \lambda = \frac{2L}{3}$ from $V = \lambda f \Rightarrow f = \frac{v}{\lambda}$ so $f = \frac{3V}{2L} = \frac{3X340}{2X0.5} = 1020Hz$

ii. for closed pipe, the third harmonic;

$L = \frac{5}{4}\lambda \Rightarrow \lambda = \frac{4L}{5}$, from $f = \frac{V}{\lambda}$ $\Rightarrow f = \frac{5V}{4L} = \frac{5X340}{4X0.5} = 850Hz$

(6). An open pipe of 30cm long and a closed pipe of 23cm long, both of the same diameter are each sounding their first overtone, and these are in union. What is the end correction of the pipe.

Soln.

For open pipe, first overtone (second harmonic)

$L + C = \lambda$ but from $V = \lambda f$, $\Rightarrow \lambda = \frac{V}{f}$.; $\frac{V}{f} = L_1 + 2C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(i)$

For closed pipe, the first overtone (second harmonic)

$L + C = \frac{3}{4}\lambda$, but $\lambda = \frac{V}{f}$ $\Rightarrow \frac{V}{f} = \frac{4}{3}(L_2 + C) \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(ii)$

Equating (i) and (ii), $C = \frac{(4X23)-(3X30)}{2} = 1cm$

(7). Two open pipes of length 92cm and 93cm respectively, give beats of frequency 3Hz when sounding their fundamental notes together. If the end corrections are 1.5cm and 1.8cm respectively. calculate the velocity of sound in air

Soln; $f_{0_1} = \frac{V}{2(L_1+2C_1)}$ and $f_{0_2} = \frac{V}{2(L_2+2c_2)}$, but $f_b = f_{0_1} - f_{0_2}$ so V =344.14ms^{-1}

NOTE; Different instruments produce different number of overtones. The number of overtones produced affect the quality of the note played. Hence the quality of the notes played by different instruments are different.

QN. Explain why a musical note played on one instrument sounds different from the same note played on another instrument.

An experiment to measure the velocity of sound in air using a resonance tube and a tuning fork of a known frequency.

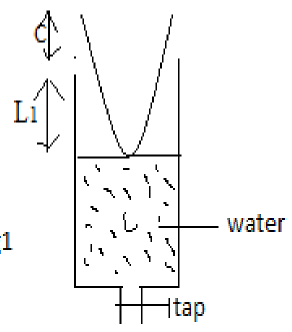


fig1

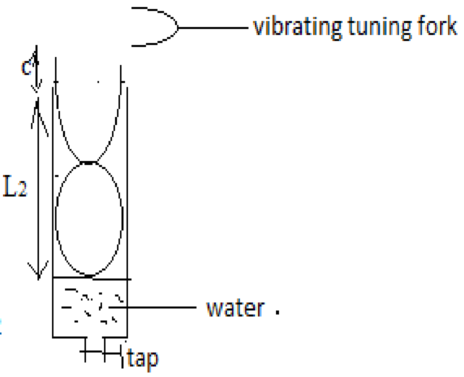


fig2

A long glass tube which can be drained from the bottom is filled with water. A vibrating tuning fork of known frequency, f is brought near the mouth of the tube. The water is then slowly drained until a loud sound is heard. The tap is then closed and the length L_1 ,of the air column at this instant is measured as in fig1. $L_1+C = \frac{\lambda}{4}$, ...(i)

The tuning fork is sounded again at the mouth of the tube and water is drained further until a loud sound is heard again. The tap is closed and the length of the air column L_2 is measured as in fig2. $L_2+C = \frac{3}{4}\lambda$, ...(ii)

(i) – (ii), $(L_2+C) - (L_1+C) = \frac{3}{4}\lambda - \frac{\lambda}{4}$ $\Rightarrow L_2 - L_1 = \frac{\lambda}{2}$, $\Rightarrow \lambda = 2(L_2-L_1)$ But from $v = \lambda f$, $\Rightarrow V = 2f(L_2 - L_1)$, Since L_1 ,L_2 and f are known, V can be calculated.

Experiment to measure the end correction using a resonance tube and a tuning fork of known frequency

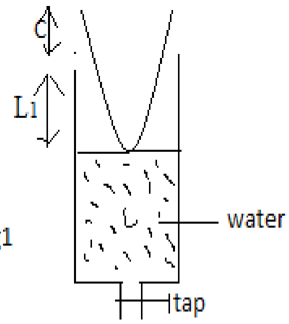


fig1

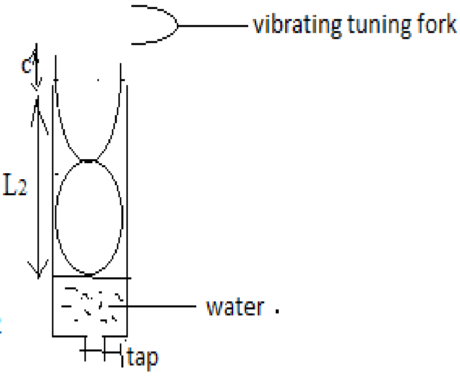


fig2

A long glass tube which can be drained from the bottom is filled with water. A vibrating tuning fork of known frequency, f is brought near the mouth of the tube. The water is then slowly drained until a loud sound is heard. The tap is then closed and the length L_1 ,of the air column at this instant is measured as in fig1. $L_1+C = \frac{\lambda}{4}$, ...(i)

The tuning fork is sounded again at the mouth of the tube and water is drained further until a loud sound is heard again. The tap is closed and the length of the air column L_2 is measured as in fig2. $L_2+C = \frac{3}{4}\lambda$, ...(ii) . Multiplying both sides of equation (i) by 3 gives; $3L_1+3C = \frac{3}{4}\lambda$, ... (iii) eqn (iii) – eqn (ii) gives

$(3L_1+3C) - (L_2+C) = \frac{3}{4}\lambda - \frac{3}{4}\lambda$ $\Rightarrow 3L_1 - L_2 = -2C$ So $C = \frac{1}{2}(L_2 - 3L_1$

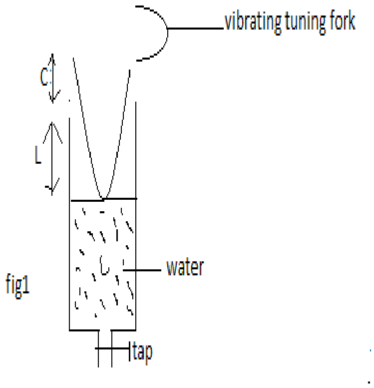
EXAMPLES

(1) A tuning fork of frequency 256Hz produces resonance in a tube of length 32.5cm and also in the one of length 95c. Calculate the speed of sound in air in air column of the tube

Soln. $V = 2f(L_2 - L_1)$ $\Rightarrow V = 2x256x(0.95-0.325)$, $V = 320ms^{-1}$

(2) A uniform tube 50cm long is filled with water and a vibrating tuning fork of frequency 512Hz is sounded and held above the tube. When the level of water os gradually lowered, the air column resonate with the tuning fork when its length is 12cm and again when it is 43.3cm. calculate
   (i) Speed of sound Ans($322.5ms^{-1}$)
   (ii) The end correction Ans (3.67cm)
   (iii) Lowest frequency to which the air can resonate if the tube is empty Ans(281.27Hz)

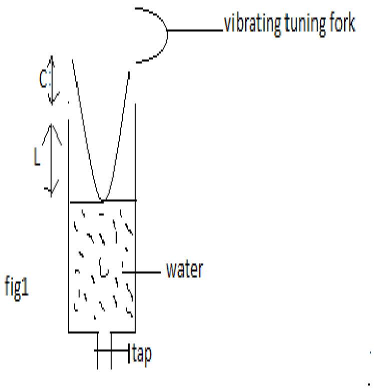
An experiment to measure the velocity of sound in air using resonance tube different tuning forks of known frequencies



A glass tube which can be drained from the bottom is filled with water. A vibrating tuning fork of known frequency, f is brought to the mouth of the tube. The water is then slowly drained until a loud sound is heard. The tap is closed and the length air column L is measured. The experiment is repeated with other tuning forks and the values of L and f recorded. The results are tabulated including values of $\frac{1}{f}$. A graph of L against $\frac{1}{f}$ is plotted and the slope,S is obtained. The velocity of sound is obtained from V = 4S

Theory ; $L + C = \frac{\lambda}{4}$, $L = \frac{\lambda}{4} - C$ But $\lambda = \frac{V}{f}$ $\Rightarrow L = \frac{1}{4f}V$

An experiment to measure the end correction using a resonance tube and different tuning forks of known frequencies



A glass tube which can be drained from the bottom is filled with water. A vibrating tuning fork of known frequency, f is brought to the mouth of the tube. The water is then slowly drained until a loud sound is heard. The tap is closed and the length air column L is measured. The experiment is repeated with other tuning forks and the values of L and f recorded. The results are tabulated including values of $\frac{1}{f}$. A graph of L against $\frac{1}{f}$ is plotted and the intercept, c of the L axis is determined from the graph. This intercept, c is the end correction

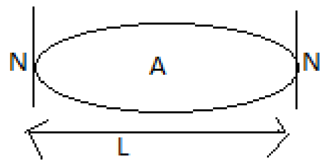
WAVES IN A STRETCHED STRING

When a stretched string is plucked, a progressive wave is formed and it travels to both ends which are fixed and the waves are reflected back to meet the incident wave. The incident and reflected waves both have the same speed, frequency and amplitude and when superimpose, a stationary wave is formed.

Modes of vibration in a stretched string

When a string is plucked in the middle, the waves below are produced

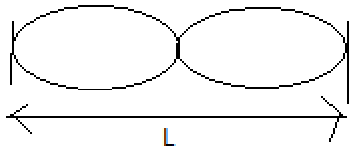
First harmonic (fundamental frequency)



L = $\frac{\lambda}{2}$ ⇒ λ = 2L, from V = λf_0, $f_0 = \frac{V}{2L}$

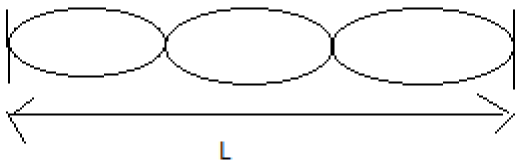
A is antinode; this is a point on a stationary wave where particles have maximum displacement. **N is a node;** is a point on a stationary wave in which particles are always at rest (zero displacement).

Second harmonic (first overtone)



L = λ, from V = λf_1 ⇒ $f_1 = \frac{V}{L}$, ⇒$f_1 = 2(\frac{V}{2L})$ ⇒$f_1 = 2f_0$

Third harmonic (second overtone)



L = $\frac{3}{2}\lambda$, ⇒ λ = $\frac{2}{3}L$; From V = λf_2 ⇒ $f_2 = \frac{V}{\lambda} = \frac{3V}{2L}$, ⇒ $f_2 = 3f_0$

Generally, $f_n = nf_0$ since $f_0 = \frac{V}{2L}$ ⇒ $f_n = \frac{nV}{2L}$, n = 1, 2, 3, 4, ...n^{th} harmonic

The velocity of a transverse wave along a stretched string.

If the string has no stiffness (it is perfectly flexible) then the velocity of a wave along a stretched string depends on;

(i) Tension, T
(ii) Mass, m
(iii) Length, l

⇒ V ∝ $T^x m^y l^z$, ⇒ V ∝ $KT^x m^y l^z$, $[v] = K[T]^x[m]^y[l]^z$, $LT^{-1} = (MLT^{-2})^x(M)^yL^z$. Equating the indices (powers) of M,L and T on both sides $x = \frac{1}{2}$, $y = \frac{-1}{2}$ and $z = \frac{1}{2}$ and from a regid mathematical investigation, K = 1

V = $K\,T^{\frac{1}{2}}\,m^{\frac{-1}{2}}\,l^{\frac{1}{2}}$ ⇒ V = $\sqrt{\frac{Tl}{m}}$ ⇒ V = $\sqrt{\frac{T}{(m/l)}}$, since $\frac{m}{l}$ is mass per unit length of the string, it follows that V = $\sqrt{\frac{T}{\mu}}$, where μ is mass per unit length .

For fundamental note (first harmonic), f = $\frac{V}{2l}$ ⇒ V = $\frac{1}{2l}\sqrt{\frac{T}{\mu}}$ since V = $\sqrt{\frac{T}{\mu}}$

Generally $f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$,n = 1, 2, 3, 4, ... n^{th} harmonic

Examples

(1) A string of length 0.5m has a mass of 5g. The string is stretched between two fixed points and plucked. If the tension is 100N, find the frequency of the second harmonic

Soln; V = $\frac{T}{\mu}$ $\mu = \frac{m}{l} = \frac{5x10^{-3}}{0.5} = 0.01kgm^{-1}$, V = $\sqrt{\frac{100}{0.01}}$ = $100ms^{-1}$

V = λf_0, ⇒ $f_0 = \frac{V}{2l} = \frac{100}{2x0.5} = 100Hz$, $f_2 = 2f_0$ = 2x100 ⇒ f_2 = 200Hz

(2) A wire under a tension of 20N is plucked at the middle to produce a note of frequency 100Hz. Calculate the;
   (i) Diameter of the wire if it's length is 1m and has a density of $600kgm^{-3}$.
   (ii) Frequency of the first overtone.

Soln; V = $\sqrt{\frac{T}{\mu}}$ and from $f_0 = \frac{V}{2l}$, ⇒ $f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$, $\mu = \frac{m}{l} = \frac{m}{1}$, ⇒ 100 = $\sqrt{\frac{20}{(m/1)}}$

⇒ 100 = $\sqrt{\frac{20x1}{m}}$, ⇒ m = 0.0005kg

$\rho = \frac{mass}{volume}$ ⇒ volume = $\frac{mass}{volume} = \frac{0.0005}{600} = 8.33x10^{-1}m^3$. But volume = $Al = \pi r^2 l$

$r = \sqrt{\frac{8.33x10^{-7}}{\pi x1}}$, r = $5.15x10^{-4}$m but d = 2r =2x$5.15x10^{-4}$ =$1.03x10^{-3}$m.

(3) A stretched wire of length 0.75m, radius of 1.36mm and density $1380kgm^{-3}$ is clamped at both ends and plucked in the middle. The fundamental note produced by the wire has the same frequency as the first overtone in the pipe of length 0.15mclosed at one end.
   (i) Sketch the stationary wave pattern in the pipe
   (ii) Calculate the tension in the wire (speed of sound in air = $330ms^{-1}$)

Soln; for the wire at fundamental note, V = $\sqrt{\frac{T}{\mu}}$, from $f_0 = \frac{V}{2l}$ ⇒ $f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ But $\mu = \frac{m}{l}$

m = ρv , ⇒ m = $\rho Al = \rho\pi r^2 l$, so $f_0 = \frac{1}{2l}\sqrt{\frac{T}{(\frac{\rho\pi r^2 l}{l})}}$,

so $f_0 = \frac{1}{2x0.75}\sqrt{\frac{T}{(\pi(1.36x10^{-3})^2x1380}}$ .... (i)

V =λf_1 , $f_1 = \frac{v}{\lambda}$, $\lambda = \frac{4}{3}l$, so $f_1 = \frac{3V}{4l}$, $f_1 = \frac{3x330}{4x0.15}$ ....(ii)

Equating (i) and (ii)

$\frac{1}{2x0.75}\sqrt{\frac{T}{(\pi(1.36x10^{-3})^2x1380}} = \frac{3x330}{4x0.15}$ ⇒ T = $4.91x10^4$N

(4) A wire of length 400mm and mass $1.2x10^{-3}$kg is under a tension of 120N. What is
  (a) The fundamental frequency of vibration
  (b) The frequency of the third harmonic

Soln ;$l = 400mm = 0.4m, m = 1.2x10^{-3}kg$ but $\mu = \frac{m}{l} = \frac{1.2x10^{-3}}{0.4} = 3x10^{-3}kgm^{-1}$ .from

$f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$, for fundamental frequency, n = 1, so $f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2x0.4}\sqrt{\frac{120}{3x10^{-3}}}$ = 250Hz

(b). For third harmonic, n = 3 , $f_3 = 3f_1$ = 3x250 = 750Hz

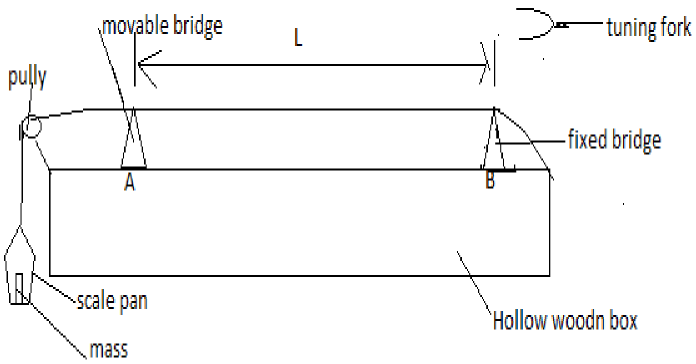
NOTE; if E is young's modulus,ρ is the density of the wire , then $f = \frac{V}{\lambda}$, but for fundamental note, $\lambda = 2l$ ⇒ $f = \frac{V}{2l}$, ⇒ $f = \frac{1}{2l}\sqrt{\frac{E}{\rho}}$ where $v = \sqrt{\frac{E}{\rho}}$

Experiment to verify f = $\frac{1}{2l}\sqrt{\frac{T}{\mu}}$

The frequency, f of the fundamental mode of vibration of a stretched string depends on the following

(i) $f \propto \frac{1}{l}$ if T and μ are constant
(ii) $f \propto \sqrt{T}$ if l and μ are constant
(iii) $f \propto \frac{1}{\sqrt{\mu}}$ if l and T are constant.

These are called laws of vibration of a fixed string. To verify these laws, we use a sonometer.



An experiment to verify that $f \propto \frac{1}{l}$

An experiment is set up as above. The mass m is kept constant so that T is constant. The string is plucked in the middle and the vibrating tuning fork of known frequency, f is placed near it. The bridge A is moved towards B until when a loud sound is heard. The distance l

between the bridges is measured and recorded together with the frequency of the fork. The experiment is repeated with different tuning forks of different frequencies and the corresponding length, l also measured. The results are tabulated including values of $\frac{1}{l}$. A graph of f against $\frac{1}{l}$ is plotted and it's a straight line through the origin implying $f \propto \frac{1}{l}$

An experiment to show how the frequency of a stretched string varies with tension

($f \propto \sqrt{T}$)

The experiment is set up as shown above. The length l between the two bridges is kept constant. A suitable mass m is attached to the free end of the string (scale pan). The string is plucked in the middle and a tuning fork of known frequency is sounded near it. The mass in the scale pan is varied until a loud sound is heard. The mass and the frequency are measured and recorded. The procedures are repeated with other forks of different frequencies. They are tabulated including values of, f^2. A graph of f^2 against m is plotted which gives a straight-line graph passing through the origin. Implying that $f^2 \propto m$. But T = mg. $\Rightarrow$ T $\propto$ m. Therefore $f^2 \propto T \Rightarrow f = \sqrt{T}$. Where, T is the tension

Experiment to show how the frequency of a stretched string varies with mass per unit length ($f \propto \frac{1}{\sqrt{\mu}}$).

The experiment is set up as shown above. The mass per unit length μ is determined by weighing. The tension is kept constant. The string is plucked in the middle and a tuning fork of known frequency f is placed near it. Bridge A is moved towards B until a loud sound heard. The distance l between the two bridges is measured and recorded. The procedures are repeated with wires of different mass per unit length. Each wire is kept under the same tension as the first wire and the tuning fork used in the first wire is not changed (is used throughout in this experiment). The results are tabulated including values of $\frac{1}{l}$ and $\frac{1}{\sqrt{\mu}}$. A graph of $\frac{1}{l}$ against $\frac{1}{\sqrt{\mu}}$ is plotted and a straight line passing through the origin is obtained. This implies $\frac{1}{l} \propto \frac{1}{\sqrt{\mu}}$ and since $f \propto \frac{1}{l} \Rightarrow f \propto \frac{1}{\sqrt{\mu}}$

Experiment to show that the wire under tension vibrate with more than one frequency.

The experiment is set up as shown above. The wire is set under tension by putting weights on the mass hanger. The wire is then plucked in the middle. Different tuning forks of different frequencies are then brought near the wire in turn to find out the one that resonate with it. The frequency f of the resonating fork is noted. Keeping l and T constant, the procedures are repeated with the string plucked at a distance of $\frac{1}{4}l \; and \; \frac{1}{8}l$ from bridge A. It is found that in each case, the frequency of the resonating fork will be different. Hence the wire under tension vibrates with more than one frequency.

EXAMPLES

(1) A sonometer wire of length 76cm is maintained under a tension of 40N and an a.c is passed through the wire. A horse shoe magnet is placed with its poles above and below the wire at its mid-point and the resultant force sets it into resonant vibration. If

the density of the material of the wire is 880kgm^{-3} and the diameter of the wire is 1mm. Find the frequency of the a.c.

Soln; $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$. ,l = 0.76m , T = 40N and μ = mass per unit length .

mass of 1m length of the wire = volume x density = $\pi r^2 x1x8800$. Where r is the radius of the wire = 0.5x10^{-3}m $\Rightarrow f = \frac{1}{2x0.76}\sqrt{\frac{40}{3.14x(0.5x10^{-3})^2x1x8800}}$ $\Rightarrow f = 49.6Hz$

(2) A weight is hung on the wire of a sonometer. When a vibrating length of the wire is adjusted to 80cm, the length it emits when plucked in the middle is in tune with a sounding fork. On adding a further weight of 100g, the vibrating length has to be altered by 1cm in order to restore the tuning. What is the initial weight on the wire.

Soln; from $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$, $\Rightarrow f = \frac{1}{2x0.8}\sqrt{\frac{T}{\mu}}$ .... (i)

When the mass of 100g is added, $T = \frac{100}{1000}9.81 = 0.981N$, new tension = T + 0.981

and new length = 0.81m $\Rightarrow f = \frac{1}{2x0.81}\sqrt{\frac{T+0.981}{\mu}}$ .... (ii)

Equating (i) and (ii) gives $\frac{1}{2x0.8}\sqrt{\frac{T}{\mu}} = \frac{1}{2x0.81}\sqrt{\frac{T+0.981}{\mu}}$; T = 37.0N,

initial weight = 37.0N

(3) The mass of a vibrating length of a sonometer wire is 1.20g and it's found that a note of frequency 512Hzis produced when the wire is sounding its second overtone. If the tension in the wire is 100N, calculate the vibrating length of the wire.

Soln; second overtone (third harmonic)

$f_2 = 3f_0 \Rightarrow f_2 = \frac{3V}{2l}$, therefore $f_2 = \frac{3}{2l}\sqrt{\frac{T}{\mu}}$ $\Rightarrow 512 = \frac{3}{2l}\sqrt{\frac{100}{(\frac{1.2x10^{-3}}{1})}}$, $l =$ 0.175m